Generalized quantifiers

Last time we noted that while in logic, quantifiers are unary: \( \forall x. \phi \), in natural languages they tend to be binary: \( \forall x : \theta. \phi \). While this makes little difference for “every” where “every girl walks” can be written either as \( \forall x. \text{girl}(x) \rightarrow \text{walk}(x) \) or \( \forall x : \text{girl}(x). \text{walk}(x) \). However, this does not work for “most”: \( \text{Most } x. \text{girl}(x) \rightarrow \text{walk}(x) \) is not the same as \( \text{Most } x : \text{girl}(x). \text{walk}(x) \).

We can look at the interpretation of such quantifiers as a binary relation between subsets:

**Definition 1** A determiner is a function \( D \) in a model \( M = \langle E, \ldots \rangle \) assigning to the domain of entities \( E \) a binary relation \( D_E \) between subsets \( A \) and \( B \).

For example we can define the meaning of every as the relation \( D_\forall \) such that \( A D_\forall B \iff A \subseteq B \). Similarly the meaning of exists is the relation \( D_\exists \) such that \( A D_\exists B \iff A \cap B \neq \emptyset \). It is easy to characterize exactly \( n \), at least \( n \), and at most \( n \) in this way.

Most is a bit trickier: The meaning of most is the relation \( D_{\text{most}} \) such that \( A D_{\text{most}} B \iff |A \cap B| \geq |A|/2 \).

It is useful to see what properties that generalized quantifiers satisfy and if differences in their properties explain linguistic differences between them.

Here are some properties that people often assert for quantifiers:

**Conservativity** If \( A, B \subseteq E \) then \( A D_E B \) if \( A \cap B D_E B \). We say that \( A \) lives on \( B \).

**Extension** If \( A, B \subseteq E \subseteq E' \) then \( A D_E B \) if \( A D_{E'} B \).

**Quantity** If \( F \) is a bijection from \( M_1 \) to \( M_2 \) then \( A D_{E_1} B \) if \( F(A) D_{E_2} F(B) \).

**Variation** For each domain \( E \) there is a domain \( E' \) such that \( E \subseteq E' \), \( A, B \), and \( C \subseteq E' \) such that \( A D_{E'} B \) and not \( A D_{E'} C \).

Conservativity says that the quantifier “lives on” the first domain. I.e., elements not in the domain make no difference in the truth of the quantifier. This is clearly true for “every” and “a”, for example. On the other hand it fails for “only”. The meaning of only girls pet Rover depends on more than just the value of pet Rover on girls.

Extension says that elements outside of \( A \) and \( B \) (the meaning of the common noun and verb phrase) make no difference to the truth of the quantifier. Again this is clearly true for “every” and “a”, but probably not for “many” and “few”. “Many girls pet Rover” may be considered to fail if there are only a few girls, but many more boys petting Rover.

Quantity simply says that the interpretation of the quantifier depends only on the relative sizes of the sets and not their identity. While this is true for most quantifiers above, there are others like “John’s” (as in John’s book) where the identity of the elements matters.

Variation just rules out trivial quantifiers that relate everything or nothing.

Most quantifiers that we will consider will satisfy all of these properties. As a result, we will not consider “only” a quantifier.

Here is an even more useful characterization of quantifiers that will help us make inferences about statements involving the quantifiers.

Relation \( D \) is
Left monotone increasing iff \( A \cap B \subseteq A' \cap B \). If several short girls are petting Rover then several girls are petting Rover. These are sometimes called persistent quantifiers.

Left monotone decreasing iff \( A \cap B \subseteq A' \cap B \). If every girl pets Rover then every red-headed girl pets Rover. These are sometimes called anti-persistent quantifiers.

Right monotone increasing iff \( A \cup B \subseteq A' \cup B \). If every girl pets Rover gently then every girl pets Rover.

Right monotone decreasing iff \( A \cup B \subseteq A' \cup B \). If no girls pet Rover then no girls pet Rover gently.

We can classify quantifiers by their properties

- “every/all”: Left monotone decreasing, right monotone increasing.
- “a”: Left monotone increasing, right monotone increasing
- “no”, “few”: Left monotone decreasing, right monotone decreasing
- “not every/all”: Left monotone increasing, right monotone decreasing.

Other quantifiers behave similarly. It is interesting to see how this classification predicts linguistic constraints:

Exercises:

1. Notice the following sentences involving “there”:
   - There are some apples in my pocket.
   - *There is every apple in my pocket.

Classify the quantifiers that do and do not work in these contexts.

2. The adverb “ever” is an example of a negative polarity item (NPI), because it seems to require a negative environment.
   - Jane hasn’t ever petted a dog.
   - *Jane has ever petted a dog.

However, sometimes a “not” is not necessary to license “ever”. E.g.,
   - Few people ever petted a dog.
   - Every one who ever petted the dog liked it.

Which properties of the determiners correlates with the distribution of ever?