Plurals

The behavior of plurals can be tricky. Consider the two sentences below:

- John and Bill walk.
- John and Bill meet.

The first of these is semantically equivalent to “John walks and Bill walks”, but the second is not equivalent to “John meets and Bill meets”. Our discussion in the previous section works fine for the first example, but not for the second. Of course similar problems arise for plurals: “The boys walk” and “The boys meet”. Our discussion of this issue follows the work of Link (1983).

We distinguish between distributive predicates like “walks” and collective predicates like “meet”. The distributive predicates are handled nicely by our previous work, but the collective predicates are not. Instead we will use lattices (or actually upper semi-lattices) to try to come up with a solution that words for both.

An i-join semilattice is a structure \(<A, \lor>\) with a complete* join operation. Complete* means that the structure is closed under all non-empty joins. You can construct examples simply by taking a regular lattice and just throwing away the bottom element. However, for our applications, we will not need the meet operation \(\land\) or negation. Instead we will be interested in i-join semilattices that are generated by a collection of individuals. That is the set of individuals will play the role of atoms (we actually call them atom* because there is no bottom element).

The language we will be dealing with includes the usual terms of first-order logic, including constants and variables and the logical operations. We also include two new term-building operations: + and \(\sigma\). Plus will be used in making plurals, while \(\sigma\) will represent “the”. The language also contains \(\leq\) and two unary predicates: IND (the individual level predicates) and COL (the set of collective predicates). There is also a special predicate AT in IND. AT picks out the atoms of our universe. Finally there are two operations on predicates: ↑ and ↓.

TERM is the smallest set such that

1. \(\text{VAR} \cup \text{CON} \subset \text{TERM}\).
2. If \(t, t' \in \text{TERM}\), then \(t + t' \in \text{TERM}\).
3. If \(x \in \text{VAR}\) and \(P \in \text{PRED}\), then \(\sigma x. P(x) \in \text{TERM}\).

PRED is the smallest set such that

1. \(\text{IND}, \text{COL} \in \text{PRED}\).
2. If \(P \in \text{PRED}\), the \(\uparrow P, \downarrow P \in \text{PRED}\).
3. If \(x \in \text{VAR}\) and \(\phi \in \text{FORM}\) then \(\lambda x. \sigma \in \text{PRED}\).
FORM is the smallest set such that

1. If $t \in \text{TERM}$ and $P \in \text{PRED}$ then $P(t) \in \text{FORM}$.

2. If $\phi, \psi \in \text{FORM}$ and $x \in \text{VAR}$, then $\neg \phi, \phi \land \psi,$ and $\phi \lor \psi \in \text{FORM}$.

3. If $t, t' \in \text{TERM}$ then $t \leq t' \in \text{FORM}$.

A model $M$ for $L$ is a triple $\langle \langle A, \land \rangle, *, I \rangle$ where

1. $\langle A, \lor \rangle$ is a free i-join semilattice, generated by a set of atoms, $AT$.

Let $PL = A - AT$.

2. $* \notin A$ is the undefined (error) element. Define $A* = A \cup \{ * \}$.

3. $I$ is an interpretation function for the non-logical constants s.t.

   - if $c \in \text{CON}$, then $I(c) \in A*$.
   - If $P \in \text{IND}$, then $I(P) \subseteq AT$.
   - if $P \in \text{COL}$, then $I(P) \subseteq PL$.

As usual we let $g$ denote a function assigning values to free variables. Meanings are assigned as follows:

**TERM:**

1. $I_g(c) = I(c)$.

2. $I_g(x) = g(X)$.

3. $I_g(t + t') = I_g(t) \lor I_g(t')$, if both $I_g(t), I_g(t') \in A$, and $*$ otherwise.

4. $I_g(\sigma x. P(x)) = \lor(I_g(P))$, if $\lor(I_g(P)) \in I_g(P)$; $*$ otherwise.

Notice that the meaning of $\sigma x. P(x)$ (read as “the P”) is the maximum element of $P$, if it exists, and is otherwise undefined. This makes sense for “the boy” if there is only one and “the boys” if the semilattice has an element representing all of the boys.

**PRED:**

1. $I_g(P) = I(P)$ for all $P$. $I_g(\leq) = \leq$ where $x \leq y$ is defined as $x \lor y = y$.

   $I_g(AT) = AT$.

2. $I_g(\lambda x. \phi) = \{ d \in A : I_{g[d/x]}(\phi) = \text{true} \}$.

3. $I_g(\uparrow P) = \text{elements of the i-join sublattice generated by } I_g(P) \text{ under } \lor$.

4. $I_g(\downarrow P) = \{ d \in AT : d \in I_g(P) \}$

Notice that if $P$ is “boy,” an individual predicate (IND) then it only holds of individuals, but $\uparrow$ boy holds of plurals. Therefore we represent “the boy” as $\sigma x. \text{boy}(x)$, while “the boys” is represented as $\sigma x. \uparrow \text{boy}(x)$.

Notice as well that “the boy” only makes sense if there is only one boy, as otherwise $\lor$ would result in a non-individual as the maximal element, and that wouldn’t make “boy” true. On the other hand, “the boys” makes sense as long as there is at least one boy.

In fact for individual predicates, we get $P(a) \land P(b)$ iff $\uparrow P(a + b)$ by the definition of $\uparrow$.

Note that for individual predicates, $\uparrow \uparrow P = P$. 

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FORM:

1. $I_g(P(t)) = true$ iff $I_g(t) \in I_g(P)$.
2. $I_g(t \leq t') = true$ iff $I_g(t) \leq I_g(t')$.
3. $I_g(\neg P) = true$ iff $I_g(P) = false$.
4. $I_g(\exists \phi) = true$ iff for some $d \in A, I_g[d/x](\phi) = true$.
5. $I_g(\phi \lor \psi) = true$ iff $I_g(\phi) = true$ or $I_g(\psi) = true$.

Now we have a way of making sense of distributive and collective verbs. ↑ allows us to group several individuals into a single collection to which we can apply “the”. With individual predicates, applying them to a collection is equivalent to applying them to each of the individuals. On the other hand, collective predicates can only be applied to non-atoms and do not distribute.

Handling Ambiguity

We talked briefly about Montague’s technique for handling ambiguity by rewriting syntax trees.