A quick review

Recall from last time that we defined a translation from CAT to types, T, as follows:

**Definition 1** $f$ is a function from CAT to $T$ such that

1. $f(S) = t$
2. $f(CN) = f(IV) = \langle e, t \rangle$
3. $f(A/B) = \langle \langle s, f(B) \rangle, f(A) \rangle$.

While CN and IV have the same type, they represent different categories. Notice that functorial categories are represented as intentional types. The reason is that they can occur in intensional contexts. Examples include:

- Jane seeks Rover.
- John believes that Jane seeks Rover.
- Jane is trying to go to the store.

As a result we must first calculate meanings as intensions and then obtain the extension when needed.

**Translating expressions**

To obtain meanings, we need to provide the lexical semantics for the languages. We assume we are given a function $g$ to map from syntax to semantics as follows:

1. $g$ is a function from $B_A$ (with the exception of $B_T$ – noun phrases, be, and necessarily) to $CON^T_{f(A)}$.
2. If $\alpha \neq \beta$ then $g(\alpha) \neq g(\beta)$.

We use the following notation for meaning. If Jane is a basic term then we indicate the meaning of Jane, $g(\text{Jane})$, by $\text{jane}$.

The translation of terms proceeds as earlier in the term, except that we must take into consideration intensions. The first rule is simple except for the fact that noun phrases are omitted:

**T1a**: If $\alpha$ is in the domain of $g$, then $\alpha$ translates into $g(\alpha)$.

From now on, we will write this translation using $\mapsto$. For example, $\alpha \mapsto g(\alpha)$.

We will come back later to see how we interpret names like Jane and John. However, first we look at terms of CN and determiners. Recall that $f(T) = f(S/IV) = \langle \langle s, f(IV) \rangle, f(S) \rangle = \langle \langle s, \langle e, t \rangle \rangle, t \rangle$. Thus terms of type $T$ will be represented as functions that take intentions of unary relations on $e$. Let’s look at how we might expect to interpret *every man*:

$$\lambda X : \langle s, \langle e, t \rangle \rangle. \forall x : e. (\text{MAN}(x) \rightarrow ^\forall X(x))$$

Because this has type $\langle \langle s, \langle e, t \rangle \rangle, t \rangle$, we see that it tells of a property of individuals, whether that property holds of all men. I.e., it determines whether or not a property holds of all men (in this world).

Because $X$ ranges over intensional unary relations in this formula, $^\forall$ must be placed before $X(x)$. We will reserve the use of lower case variable symbols for elements of $e$, and won’t mark their use with
a type from now on. Similarly, capitalized variables will stand for elements of type \( \langle s, \langle e, t \rangle \rangle \), that is intensions of unary predicates.

Because we must apply this term to an intension, we won’t be surprised to see we need to pass in the intension of walk \( \land \) \( \text{WALK} \), rather than its extension. When we apply the meaning of \textit{every man} to the intension of \( \text{walk} \) we get:

\[
\forall x. (\text{MAN}(x) \rightarrow \land \land \text{WALK}(x))
\]

but by \( \land \land \) elimination, this reduces to

\[
\forall x. (\text{MAN}(x) \rightarrow \text{WALK}(x))
\]

From this we can see that the translation of \textit{every} should be:

\[
g(\text{every}) = \lambda Y. \lambda X. \forall x. (\land \land Y(x) \rightarrow \land \land X(x))
\]

Notice that the first parameter, \( Y \), also expects an intension, and hence uses the \( \land \) operator to obtain the extension of the term.

The meanings for other determiners is similar. For example,

\[
g(a) = \lambda Y. \lambda X. \exists x. (\land \land Y(x) \land \land X(x))
\]

\[
g(\text{the}) = \lambda Y. \lambda X. \exists x. \forall y. (\land \land Y(y) \leftrightarrow x = y) \land \land X(x))
\]

Notice that existence and uniqueness are asserted in the meaning of \textit{the}, rather than presupposed. Alternatives are possible, but this is the meaning given in PTQ.

From above, we see that the type of a noun phrase like “a man” or “every girl” has type \( \langle \langle s, \langle e, t \rangle \rangle, t \rangle \rangle \).

Thus we expect proper nouns to have a similar type. This is similar to what we saw before, except that now we have intensions. Thus we will interpret names as follows:

- \( T1(b) \): \textit{Jane} \mapsto \lambda X. \land \land X(\text{jane}) \) where \text{jane} is the element of \( e \) corresponding to \textit{Jane}.
- \( \text{John} \mapsto \lambda X. \land \land X(\text{john}), \ldots \)
- \( \text{he} \) \( n \mapsto \lambda X. \land \land X(x_n) \) for all \( n \).

We’ll come back and explain later about the translation of \textit{he} \( n \).

The key idea here is that the set of properties of an individual and the individual are uniquely related:

\[
\forall x. \forall y.(x = y \leftrightarrow (\lambda X. \land \land X(x) = \lambda X. \land \land X(y)))
\]

Of course not every set of properties corresponds to an individual. E.g., the collection of properties that hold of “all men” is unlikely to hold of any individual man.