Montague grammar

We will now be repeating some of our earlier work on semantics, though generalized to handle intensional constructs. Last time we saw that we cannot define the truth of a language within that language (Tarski’s theorem on the undefinability of truth in rich enough languages), but that we can define the truth of fragments of the language within a somewhat richer language. That is what we will end up doing. We will be following the lead of Montague, whose ground breaking work during the 1970’s resulted in the foundation of modern semantics. Occasionally we will make slightly different choices in order to bring to bear more recent improvements, but the overall structure will match that of Montague’s PTQ (proper treatment of quantification) model.

Before getting started, we remind you that natural language is ambiguous. E.g., “The fat dogs and cats couldn’t run very fast” and our old familiar statement “Every child threw a ball”. In the first it is not clear whether we are talking about all cats or only fast ones, and in the second we are not sure whether every child threw the same ball or distinct ones.

Because we want our semantics to be compositional, we will need to distinguish these by providing different syntactic representations, which will occasionally require our syntactic analysis to be a bit more complex (and less obvious) than might be desired.

Do keep in mind that sentences with the same meaning may have very different syntactic forms. E.g. “John threw the ball” vs “The ball was thrown by John.” That is not a problem with our compositional analysis, whereas different meanings from the same syntactic form is a problem.

We have already seen that there are two ways of providing semantics. One is to provide semantics directly, by computing the meaning of a sentence with respect to a context (model) to be either true or false. The other, more indirect, approach that we used in following Blackburn and Bos, was to translate sentences to formulas of a logical language (there the first order predicate logic, here as an intentional predicate logic), and then interpret those statements in a model.

In this second version of semantics, the interpretation of statements of a logical language are always unambiguous. Hence the initial translation into formulas of the logical language is where the ambiguity should appear. In particular, the translation of an ambiguous statement of a natural language should result in two or more statements of the logical language.

Another feature of the indirect semantics is that we need both the translation and interpretation phases to be compositional as we want the resulting semantics to be compositional.

We will be using a rather crude syntactic analysis of expressions for our semantics. Typically one would need to go well beyond what we are doing here to extend what we are doing to a richer natural language, but it will suffice for our purposes.

We will be using a categorial grammar of the sort introduced in lecture 21.

Definition 1 CAT, the set of categories, is the smallest set such that:

1. S, CN, IV are in CAT
2. If A and B are in CAT then so is A/B.

From these we can define other categories:
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In the following, we will want to specify rules for both syntax and semantics for expressions of English. To do this, we will need to build both parse trees (or their equivalent) and semantic translations. Because we will wish to accurately recognize correct syntax (e.g., “John walks” is fine, but “John walk” is not), we will make our rules for building up expressions syntactically a bit more complex than usual.

We begin by providing mutually recursive rules for building up all of the terms of category A, denoted by $P_A$. Let $B_A$ be the collection of all lexical elements (basic expressions, typically individual words) of category A. We have listed some of these in the table above.

The first rule for defining $P_A$ for all $A$ is the following:

**S1:** $B_A$ is a subset of $P_A$ for all $A$.

Next we provide a rule that allows us to build up elements of category S:

**S2:** If $\alpha$ is in $P_T$ and $\delta$ is in $P_{IV}$, then $F_1(\alpha, \delta) = \alpha\delta'$ is in $P_S$ where $\delta'$ is the results of replacing the main verb in $\delta$ by its third-person singular present form.

Here and later we will use functions of the form $F_i$ for the rules to build up syntax. In the above rule, it is quite clear that we can put together the words “Mary” and “walk” to obtain “Mary walks” by using $F_1$. (Note that writing $\alpha\delta'$ means concatenating the two expressions into a larger expression.) However there are more complex phrases that illustrate some of the complexities: Combine “John” and “walk and try to throw the ball” to obtain “John walks and tries to throw the ball”.

Of course, it would be much more complicated with a richer language, but luckily we do not have first and second person, plurals, or past or future tense. We could add them, but they would only make our life more complex without adding key foundational ideas. Note also that we are skimming over such tasks as how one recognizes the main verb in an expression.

We differ from Montague (and the main stream of the text), by treating all determiners the same:

**S3:** If $\sigma$ is in $P_{T/CN}$ and $\tau$ in $P_{CN}$, then $F_2(\sigma, \tau) = \sigma\tau$ is in $P_T$.

Now we can draw parse trees:

```
A boy sleeps, S, S2
   /   \
A boy, T, S3'    sleep, IV
   /   \
A, T/CN     boy, CN
```

We also define a translation from CAT to types, $T$, as follows:

**Definition 2** $f$ is a function from CAT to $T$ such that
1. \( f(S) = t \)
2. \( f(CN) = f(IV) = (e, t) \)
3. \( f(A/B) = \langle \langle s, f(B) \rangle, f(A) \rangle \).