Work and Span

• With a sequential algorithm, we consider $T(n)$ as its runtime
• For a parallel algorithm, we will consider $T_P$ or $T_P(n)$ as the runtime of the algorithm using $P$ processors
• There are two important runtime quantities for a parallel algorithm:
  • How long it would take if it were to run on one processor (work)
  • How long it would take if it were as parallel as possible (span)
Definitions

• **Work**: $T_1(n) = T(n)$ or $T_1$ is how long it takes to run on one processor, that is the total of all the running times of all the pieces of the algorithm if executed sequentially.

• **Span**: $T_\infty(n)$ or $T_\infty$ is how long it takes to run on an unlimited number of processors.
  • Not necessarily $O(1)$ time
  • Still need to do forking and combine results
A program execution using `fork` and `join` can be seen as a DAG

- A DAG is a graph that is directed (edges have direction (arrows)), and those arrows do not create a cycle (path that starts and ends at the same node).

- **Nodes**: Pieces of work, typically $O(1)$ amount of work
- **Edges**: Dependencies – Source must finish before destination starts

- A `fork` “ends a node” and makes two outgoing edges
  - New thread and continuation of current thread

- A `join` “ends a node” and makes a node with two incoming edges
  - Node just ended and last node of thread joined on
We can now describe work and span as:

**Work**: How long it would take on 1 processor = $T_1$
Sum of run-time of all nodes in DAG, i.e. number of nodes

**Span**: How long it would take infinity processors = $T_\infty$
Length, i.e. number of edges in longest path in DAG

$T_\infty = 7$
The work in the nodes in the top half is to create two subproblems.

The work in the nodes in the bottom half is to combine two results.

$T_1$ is $O(n)$ since there are approximately $2n$ nodes.

$T_\infty$ is $O(\log n)$ two trees of height $\log n$ each.
Performance

- **Speedup** on $P$ processors: $\frac{T_1}{T_P}$
  - Ratio of how much faster it would run on $P$ processors
  - E.g., if $T_1$ is 20 and $T_4$ is 8, then speedup is 2.5
- **Perfect speedup**: $P$ as we vary $P$
  - E.g., 4 for the example above
  - Rare due to overhead of thread creation and communication
- **Perfect linear speedup**: doubling $P$ cuts running time in half
  - Not upper limit
Parallelism

• Reporting $T_1/T_P$ can overstate advantages of parallelism
  • $T_1$ is runtime of parallel algorithm on 1 processor
  • Likely much slower than sequential algorithm

• More realistic speedup definition $S/T_P$
  • $S$ time for sequential algorithm
  • Lower than $T_1/T_P$

• **Parallelism**: $T_1/T_\infty$
  • Maximum possible speedup
  • At least as great as speedup for any $P$
  • e.g., for our sum array problem, parallelism is $O(n/ \log n)$
  • We can hope for an exponential speedup over sequential version
ForkJoin guarantees expected bound

- \( T_P = O\left(\left(\frac{T_1}{P}\right) + T_\infty\right) \)
  - Given \( P \) processors, no framework can beat \( \frac{T_1}{P} \) or \( T_\infty \) by more than a constant factor
  - When \( P \) is small, \( \frac{T_1}{P} \) is dominant, giving roughly linear speedup
  - When \( P \) grows, limit influenced by span
- Framework on average gives best performance, assuming user did follow the paradigm as best as possible:
  - All threads ~ same work, careful with load balancing

- Bottom line:
  - Focus on your algorithms, data structures, and cut-offs rather than number of processors and scheduling.
  - Just need \( T_1, T_\infty, \) and \( P \) to analyze running time
Examples for $T_P = O((T_1/P) + T_\infty)$

• For summing:
  • $T_1 = O(n)$
  • $T_\infty = O(\log n)$
  • So expect $T_P = O\left(\frac{n}{P} + \log n\right)$

• If instead:
  • $T_1 = O(n^2)$
  • $T_\infty = O(n)$
  • Then expect $T_P = O\left(\frac{n^2}{P} + n\right)$
Amdahl’s Law

• Upper bound on speed-up!
• Suppose the work is 1 unit time.
• Let S be portion of execution that cannot be parallelized.
• $T_1 = S + (1 - S) = 1$
• Suppose we get perfect speedup on parallel portion.
  • $T_P = S + \frac{(1-S)}{P}$
• Then overall speedup with $P$ processors (Amdahl’s law):
  • $\frac{T_1}{T_P} = \frac{1}{(S + \frac{1-S}{P})}$
  • *Parallelism* (∞ processors) is: $\frac{T_1}{T_∞} = \frac{1}{S}$
Bad news

• *Parallelism* (∞ processors) is: \( \frac{T_1}{T_∞} = \frac{1}{S} \)

• If 33% of program is sequential, then absolute best speedup is \( \frac{1}{0.33} = 3 \)
  • That means infinitely many processors won’t help us get more than a 3 times speed-up!

• From 1980 - 2005, every 12 years gave 100x speedup
  • Now suppose processor speed is same but 256 processors instead of 1.
  • To get 100x speedup, need \( 100 \leq \frac{1}{(S+\frac{1-s}{P})}, \text{ P}=256 \)
  • Solve for \( S \leq 0.61\% \), so need code to be 99.4% perfectly parallel.
So let’s give up?

• Amdahl tells us that if a particular algorithm has too many sequential computations, it’s better to find a more parallelizable algorithm than to just add more processors.

• Not all is lost. We can change what we compute
  • Computer graphics now much better in video games with GPU’s -- not much faster, but much more detail.

• Side note: Moore’s law is just an observation, while Amdahl’s law is an actual mathematical theorem
Sharing resources

- We’re done talking about parallelism.
- Our goal is no longer (necessarily) “to make the program faster”.
- The ForkJoin Framework is great, but it doesn’t actually allow us to share resources.
  - Two threads only interact at birth and death
- Strategy won’t work well when:
  - Memory accessed by threads is overlapping or unpredictable
  - Threads are doing independent tasks needing access to same resources (rather than implementing the same algorithm)
- For the next few lectures, we’ll investigate what happens when we lift that restriction.
  - Two threads can run different algorithms now