

## Language acquisition

<http://www.youtube.com/watch?v=RE4ce4mexrU>

(4:30)

## LANGUAGE MODELING: SMOOTHING

David Kauchak  
CS159 – Spring 2019

some slides adapted from  
Jason Eisner

## Admin

### Assignment 2 out

- ▣ bigram language modeling
- ▣ Java
- ▣ Can work with partners
  - Anyone looking for a partner?
- ▣ 2a: Due this Friday
- ▣ 2b: Due next Friday
- ▣ Style/commenting (JavaDoc)
- ▣ Some advice
  - Start now!
  - Spend 1-2 hours working out an example by hand (you can check your answers with me)
  - HashMap

## Admin

### Our first quiz (**when?**)

- ▣ In-class (~30 min.)
- ▣ Topics
  - corpus analysis
  - regular expressions
  - probability
  - language modeling
- ▣ Open book/notes
  - we'll try it out for this one
  - better to assume closed book (30 minutes goes by fast!)
- ▣ 10% of your grade

## Admin

Lab next class

Meet in Edmunds 105

## Today



## Today

Take home ideas:

- Key idea of smoothing is to redistribute the probability to handle less seen (or never seen) events
  - Still must always maintain a true probability distribution
- Lots of ways of smoothing data
- Should take into account features in your data!

## Smoothing

What if our test set contains the following sentence, but one of the trigrams never occurred in our training data?

$P(\text{I think today is a good day to be me}) =$

$P(\text{I} \mid \langle \text{start} \rangle \langle \text{start} \rangle) \times$

$P(\text{think} \mid \langle \text{start} \rangle \text{I}) \times$

$P(\text{today} \mid \text{I think}) \times$

$P(\text{is} \mid \text{think today}) \times$

$P(\text{a} \mid \text{today is}) \times$

$P(\text{good} \mid \text{is a}) \times$

...

If any of these has never been seen before, prob = 0!

## Smoothing

$P(\text{I think today is a good day to be me}) =$

$P(\text{I} \mid \langle \text{start} \rangle \langle \text{start} \rangle) \times$

$P(\text{think} \mid \langle \text{start} \rangle \text{I}) \times$

$P(\text{today} \mid \text{I think}) \times$

$P(\text{is} \mid \text{think today}) \times$

$P(\text{a} \mid \text{today is}) \times$

$P(\text{good} \mid \text{is a}) \times$

...

These probability estimates may be inaccurate. Smoothing can help reduce some of the noise.

## The general smoothing problem

			modification	probability
see the abacus	1	1/3	?	?
see the abbot	0	0/3	?	?
see the abduct	0	0/3	?	?
see the above	2	2/3	?	?
see the Abram	0	0/3	?	?
...			?	?
see the zygote	0	0/3	?	?
Total	3	3/3	?	?

## Add-lambda smoothing

A large dictionary makes novel events too probable.

add  $\lambda = 0.01$  to all counts

see the abacus	1	1/3	1.01	1.01/203
see the abbot	0	0/3	0.01	0.01/203
see the abduct	0	0/3	0.01	0.01/203
see the above	2	2/3	2.01	2.01/203
see the Abram	0	0/3	0.01	0.01/203
...			0.01	0.01/203
see the zygote	0	0/3	0.01	0.01/203
Total	3	3/3	203	

## Add-lambda smoothing

How should we pick lambda?

see the abacus	1	1/3	1.01	1.01/203
see the abbot	0	0/3	0.01	0.01/203
see the abduct	0	0/3	0.01	0.01/203
see the above	2	2/3	2.01	2.01/203
see the Abram	0	0/3	0.01	0.01/203
...			0.01	0.01/203
see the zygote	0	0/3	0.01	0.01/203
Total	3	3/3	203	

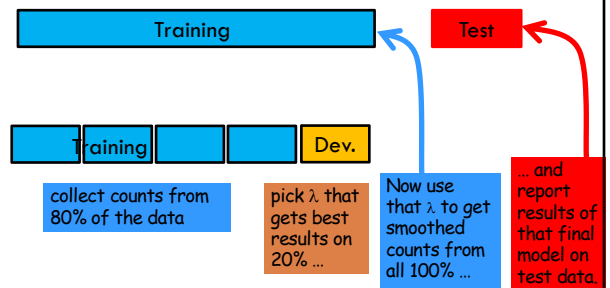
## Setting smoothing parameters

Idea 1: try many  $\lambda$  values & report the one that gets the best results?



Is this fair/appropriate?

## Setting smoothing parameters



problems? ideas?

## Vocabulary

n-gram language modeling assumes we have a fixed vocabulary

- why?

Probability distributions are over finite events!

What happens when we encounter a word not in our vocabulary (Out Of Vocabulary)?

- If we don't do anything, prob = 0
- Smoothing doesn't really help us with this!

## Vocabulary

To make this explicit, smoothing helps us with...

all entries in our vocabulary

↓

see the abacus	1	1.01
see the abbot	0	0.01
see the abduct	0	0.01
see the above	2	2.01
see the Abram	0	0.01
...		0.01
see the zygote	0	0.01

## Vocabulary

and...

Vocabulary	Counts	Smoothed counts
a	10	10.01
able	1	1.01
about	2	2.01
account	0	0.01
acid	0	0.01
across	3	3.01
...	...	...
young	1	1.01
zebra	0	0.01

How can we have words in our vocabulary we've never seen before?

## Vocabulary

Choosing a vocabulary: **ideas?**

- ▣ Grab a list of English words from somewhere
- ▣ Use all of the words in your training data
- ▣ Use some of the words in your training data
  - for example, all those that occur more than  $k$  times

Benefits/drawbacks?

- ▣ Ideally your vocabulary should represent words you're likely to see
- ▣ Too many words: end up washing out your probability estimates (and getting poor estimates)
- ▣ Too few: lots of out of vocabulary

## Vocabulary

No matter how you chose your vocabulary, you're still going to have out of vocabulary (OOV) words

How can we deal with this?

- ▣ Ignore words we've never seen before
  - Somewhat unsatisfying, though can work depending on the application
  - Probability is then dependent on how many in vocabulary words are seen in a sentence/text
- ▣ Use a special symbol for OOV words and estimate the probability of out of vocabulary

## Out of vocabulary

Add an extra word in your vocabulary to denote OOV (<OOV>, <UNK>)

Replace all words in your training corpus not in the vocabulary with <UNK>

- ▣ You'll get bigrams, trigrams, etc with <UNK>
  - $p(\text{<UNK>} \mid \text{"I am"})$
  - $p(\text{fast} \mid \text{"I <UNK>"})$

During testing, similarly replace all OOV with <UNK>

## Choosing a vocabulary

A common approach (and the one we'll use for the assignment):

- ▣ Replace the first occurrence of each word by <UNK> in a data set
- ▣ Estimate probabilities normally

Vocabulary then is all words that occurred two or more times

This also discounts all word counts by 1 and gives that probability mass to <UNK>

## Storing the table

How are we storing this table?  
Should we store all entries?

see the abacus	1	1/3	1.01	1.01/203
see the abbot	0	0/3	0.01	0.01/203
see the abduct	0	0/3	0.01	0.01/203
see the above	2	2/3	2.01	2.01/203
see the Abram	0	0/3	0.01	0.01/203
...			0.01	0.01/203
see the zygote	0	0/3	0.01	0.01/203
Total	3	3/3	203	

## Storing the table

Hashtable (e.g. HashMap)

- ▣ fast retrieval
- ▣ fairly good memory usage

Only store those entries of things we've seen

- ▣ for example, we don't store  $|V|^3$  trigrams

For trigrams we can:

- ▣ Store one hashtable with bigrams as keys
- ▣ Store a hashtable of hashtables (I'm recommending this)

## Storing the table: add-lambda smoothing

For those we've seen before:

Unsmoothed (MLE)

$$P(c|ab) = \frac{C(abc)}{C(ab)}$$

add-lambda smoothing

$$P(c|ab) = \frac{C(abc) + \lambda}{C(ab) + ?}$$

see the abacus	1	1/3	1.01	1.01/203
see the abbot	0	0/3	0.01	0.01/203
see the abduct	0	0/3	0.01	0.01/203
see the above	2	2/3	2.01	2.01/203
see the Abram	0	0/3	0.01	0.01/203
...			0.01	0.01/203
see the zygote	0	0/3	0.01	0.01/203
Total	3	3/3	203	

What value do we need here to make sure it stays a probability distribution?

### Storing the table: add-lambda smoothing

For those we've seen before:

Unsmoothed (MLE) add-lambda smoothing

$$P(c|ab) = \frac{C(abc)}{C(ab)}$$

$$P(c|ab) = \frac{C(abc) + \lambda}{C(ab) + \lambda V}$$

see the abacus	1	1/3	1.01	1.01/203
see the abbot	0	0/3	0.01	0.01/203
see the abduct	0	0/3	0.01	0.01/203
see the above	2	2/3	2.01	2.01/203
see the Abram	0	0/3	0.01	0.01/203
...			0.01	0.01/203
see the zygote	0	0/3	0.01	0.01/203
Total	3	3/3	203	

For each word in the vocabulary, we pretend we've seen it  $\lambda$  times more ( $V$  = vocabulary size).

### Storing the table: add-lambda smoothing

For those we've seen before:

$$P(c|ab) = \frac{C(abc) + \lambda}{C(ab) + \lambda V}$$

Unseen n-grams:  $p(z|ab) = ?$

$$P(z|ab) = \frac{\lambda}{C(ab) + \lambda V}$$

### Problems with frequency based smoothing

The following bigrams have never been seen:

$p(X | \text{San})$        $p(X | \text{ate})$

Which would add-lambda pick as most likely?

Which would you pick?

### Witten-Bell Discounting

Some words are more likely to be followed by new words

Diego	food
Francisco	apples
San Luis	bananas
Jose	ate hamburgers
Marcos	a lot
	for two
	grapes
	...

## Witten-Bell Discounting

Probability mass is shifted around, depending on the context of words

If  $P(w_i | w_{i-1}, \dots, w_{i-m}) = 0$ , then the smoothed probability  $P_{WB}(w_i | w_{i-1}, \dots, w_{i-m})$  is higher if the sequence  $w_{i-1}, \dots, w_{i-m}$  occurs with many different words  $w_k$

## Problems with frequency based smoothing

The following trigrams have never been seen:

$p(\text{car} | \text{see the})$

$p(\text{zygote} | \text{see the})$

$p(\text{cumquat} | \text{see the})$

Which would add-lambda pick as most likely?  
Witten-Bell?

Which would you pick?

## Better smoothing approaches

Utilize information in lower-order models

Interpolation

- Combine probabilities of lower-order models in some linear combination

Backoff

$$P(z | xy) = \begin{cases} \frac{C^*(xyz)}{C(xy)} & \text{if } C(xyz) > k \\ \alpha(xy)P(z | y) & \text{otherwise} \end{cases}$$

- Often  $k = 0$  (or 1)
- Combine the probabilities by "backing off" to lower models only when we don't have enough information

## Smoothing: simple interpolation

$$P(z | xy) \approx \lambda \frac{C(xyz)}{C(xy)} + \mu \frac{C(yz)}{C(y)} + (1 - \lambda - \mu) \frac{C(z)}{C(\bullet)}$$

Trigram is very context specific, very noisy

Unigram is context-independent, smooth

Interpolate Trigram, Bigram, Unigram for best combination

How should we determine  $\lambda$  and  $\mu$ ?



## Smoothing: finding parameter values

Just like we talked about before, split training data into training and development

Try lots of different values for  $\lambda$ ,  $\mu$  on heldout data, pick best

Two approaches for finding these efficiently

- ▣ EM (expectation maximization)
- ▣ “Powell search” – see Numerical Recipes in C

## Backoff models: absolute discounting

$$P_{\text{absolute}}(z | xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{\text{absolute}}(z | y) & \text{otherwise} \end{cases}$$

Subtract some absolute number from each of the counts (e.g. 0.75)

- ▣ How will this affect rare words?
- ▣ How will this affect common words?

## Backoff models: absolute discounting

$$P_{\text{absolute}}(z | xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{\text{absolute}}(z | y) & \text{otherwise} \end{cases}$$

Subtract some absolute number from each of the counts (e.g. 0.75)

- ▣ will have a large effect on low counts (rare words)
- ▣ will have a small effect on large counts (common words)

## Backoff models: absolute discounting

$$P_{\text{absolute}}(z | xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{\text{absolute}}(z | y) & \text{otherwise} \end{cases}$$

What is  $\alpha(xy)$ ?

### Backoff models: absolute discounting

trigram model:  $p(z|xy)$  (before discounting)   
 trigram model  $p(z|xy)$  (after discounting)   
 bigram model  $p(z|y)^*$  (\*for  $z$  where  $xyz$  didn't occur)

$$P_{absolute}(z|xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{absolute}(z|y) & \text{otherwise} \end{cases}$$

### Backoff models: absolute discounting

see the dog	1
see the cat	2
see the banana	4
see the man	1
see the woman	1
see the car	1

$p(\text{cat} | \text{see the}) = ?$   
 $p(\text{puppy} | \text{see the}) = ?$

$$P_{absolute}(z|xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{absolute}(z|y) & \text{otherwise} \end{cases}$$

### Backoff models: absolute discounting

see the dog	1	$p(\text{cat}   \text{see the}) = ?$
see the cat	2	
see the banana	4	
see the man	1	
see the woman	1	
see the car	1	

$$\frac{2 - D}{10} = \frac{2 - 0.75}{10} = .125$$

$$P_{absolute}(z|xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{absolute}(z|y) & \text{otherwise} \end{cases}$$

### Backoff models: absolute discounting

see the dog	1	$p(\text{puppy}   \text{see the}) = ?$
see the cat	2	
see the banana	4	
see the man	1	
see the woman	1	
see the car	1	

$\alpha(\text{see the}) = ?$   
 How much probability mass did we reserve/discount for the bigram model?

$$P_{absolute}(z|xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{absolute}(z|y) & \text{otherwise} \end{cases}$$

## Backoff models: absolute discounting

see the dog	1	$p(\text{puppy} \mid \text{see the}) = ?$
see the cat	2	
see the banana	4	$\alpha(\text{see the}) = ?$
see the man	1	
see the woman	1	$\frac{\# \text{ of types starting with "see the"} * D}{\text{count("see the")}}$
see the car	1	

For each of the unique trigrams, we subtracted  $D/\text{count}(\text{"see the"})$  from the probability distribution

$$P_{\text{absolute}}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{\text{absolute}}(z \mid y) & \text{otherwise} \end{cases}$$

## Backoff models: absolute discounting

see the dog	1	$p(\text{puppy} \mid \text{see the}) = ?$
see the cat	2	
see the banana	4	$\alpha(\text{see the}) = ?$
see the man	1	
see the woman	1	$\frac{\# \text{ of types starting with "see the"} * D}{\text{count("see the")}}$
see the car	1	

$$\text{reserved\_mass}(\text{see the}) = \frac{6 * D}{10} = \frac{6 * 0.75}{10} = 0.45$$

distribute this probability mass to all bigrams that we are backing off to

$$P_{\text{absolute}}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy)P_{\text{absolute}}(z \mid y) & \text{otherwise} \end{cases}$$

## Calculating $\alpha$

We have some number of bigrams we're going to backoff to, i.e. those  $X$  where  $C(\text{see the } X) = 0$ , that is unseen trigrams starting with "see the"

When we backoff, for each of these, we'll be including their probability in the model:  $P(X \mid \text{the})$

$\alpha$  is the normalizing constant so that the sum of these probabilities equals the reserved probability mass

$$\alpha(\text{see the}) * \sum_{X: C(\text{see the } X) = 0} p(X \mid \text{the}) = \text{reserved\_mass}(\text{see the})$$

## Calculating $\alpha$

We can calculate  $\alpha$  two ways

- Based on those we haven't seen:

$$\alpha(\text{see the}) = \frac{\text{reserved\_mass}(\text{see the})}{\sum_{X: C(\text{see the } X) = 0} p(X \mid \text{the})}$$

- Or, more often, based on those we do see:

$$\alpha(\text{see the}) = \frac{\text{reserved\_mass}(\text{see the})}{1 - \sum_{X: C(\text{see the } X) > 0} p(X \mid \text{the})}$$

### Calculating $\alpha$ in general: trigrams

$p(C | A B)$

Calculate the reserved mass

$$\text{reserved\_mass}(\text{bigram} \rightarrow A B) = \frac{\text{\# of types starting with bigram * D}}{\text{count}(\text{bigram})}$$

Calculate the sum of the backed off probability. For bigram "A B":

$$1 - \sum_{X:C(A B X) > 0} p(X | B) \quad \text{either is fine in practice, the left is easier} \quad \sum_{X:C(A B X) = 0} p(X | B)$$

Calculate  $\alpha$

$$\alpha(A B) = \frac{\text{reserved\_mass}(A B)}{1 - \sum_{X:C(A B X) > 0} p(X | B)}$$

1 - the sum of the bigram probabilities of those trigrams that we saw starting with bigram A B

### Calculating $\alpha$ in general: bigrams

$p(B | A)$

Calculate the reserved mass

$$\text{reserved\_mass}(\text{unigram} \rightarrow A) = \frac{\text{\# of types starting with unigram * D}}{\text{count}(\text{unigram})}$$

Calculate the sum of the backed off probability. For bigram "A B":

$$1 - \sum_{X:C(A X) > 0} p(X) \quad \text{either is fine in practice, the left is easier} \quad \sum_{X:C(A X) = 0} p(X)$$

Calculate  $\alpha$

$$\alpha(A) = \frac{\text{reserved\_mass}(A)}{1 - \sum_{X:C(A X) > 0} p(X)}$$

1 - the sum of the unigram probabilities of those bigrams that we saw starting with word A

### Calculating backoff models in practice

Store the  $\alpha$ s in another table

- If it's a trigram backed off to a bigram, it's a table keyed by the bigrams
- If it's a bigram backed off to a unigram, it's a table keyed by the unigrams

Compute the  $\alpha$ s during training

- After calculating all of the probabilities of seen unigrams/bigrams/trigrams
- Go back through and calculate the  $\alpha$ s (you should have all of the information you need)

During testing, it should then be easy to apply the backoff model with the  $\alpha$ s pre-calculated

### Backoff models: absolute discounting

the Dow Jones 10  
the Dow rose 5  
the Dow fell 5

$p(\text{jumped} | \text{the Dow}) = ?$   
What is the reserved mass?

$$\frac{\text{\# of types starting with "the Dow" * D}}{\text{count}(\text{"the Dow"})}$$

$$\text{reserved\_mass}(\text{the Dow}) = \frac{3 * D}{20} = \frac{3 * 0.75}{20} = 0.115$$

$$\alpha(\text{the Dow}) = \frac{\text{reserved\_mass}(\text{see the})}{1 - \sum_{X:C(\text{the Dow } X) > 0} p(X | \text{the})}$$

## Backoff models: absolute discounting

$$\text{reserved\_mass} = \frac{\text{\# of types starting with bigram} * D}{\text{count}(\text{bigram})}$$

Two nice attributes:

- ▣ decreases if we've seen more bigrams
  - should be more confident that the unseen trigram is no good
- ▣ increases if the bigram tends to be followed by lots of other words
  - will be more likely to see an unseen trigram