

# Search Algorithms

[ Peter Mawhorter ]

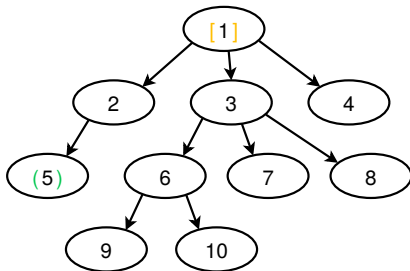
University of California Santa Cruz

March 31st, 2016

# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

- ▶ Add the start state to to\_visit.
- ▶ Repeat:
  - ▶ Take a state off the to\_visit list.
  - ▶ If it's the goal state:
    - ▶ We're done!
  - ▶ If not:
    - ▶ Add all successor states to to\_visit.



Depth-first search (DFS): to\_visit is a stack

Breadth-first search (BFS): to\_visit is a queue

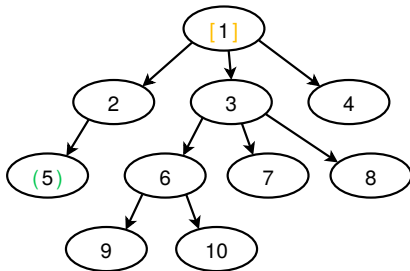
# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:

1

BFS:



Depth-first search (DFS): to\_visit is a stack

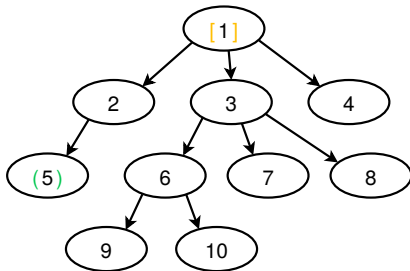
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# Review: Ordering

In what order will BFS and DFS visit these states?  
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DFS:  
1, 2, 5?

BFS:



Depth-first search (DFS): to\_visit is a stack

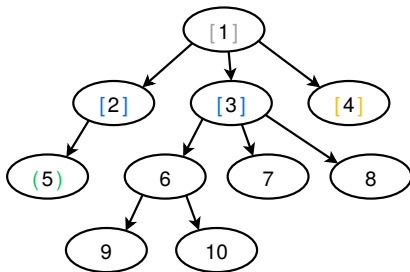
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# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4

BFS:



Depth-first search (DFS): to\_visit is a stack

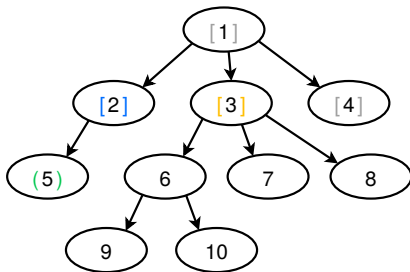
Breadth-first search (BFS): to\_visit is a queue

# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4, 3

BFS:



Depth-first search (DFS): to\_visit is a stack

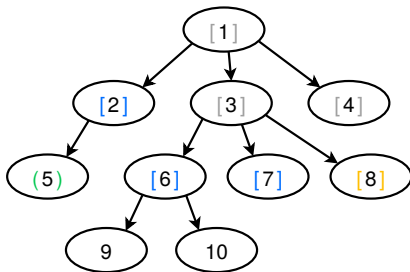
Breadth-first search (BFS): to\_visit is a queue

# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4, 3, 8

BFS:



Depth-first search (DFS): to\_visit is a stack

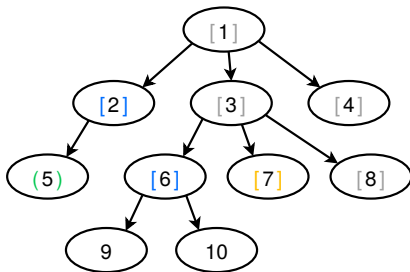
Breadth-first search (BFS): to\_visit is a queue

# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4, 3, 8, 7

BFS:



Depth-first search (DFS): to\_visit is a stack

Breadth-first search (BFS): to\_visit is a queue

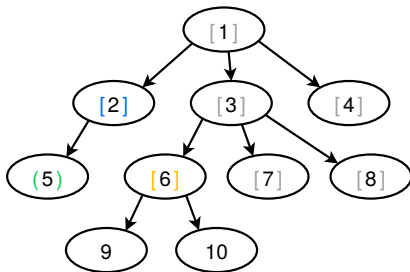


# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4, 3, 8, 7, 6

BFS:



Depth-first search (DFS): to\_visit is a stack

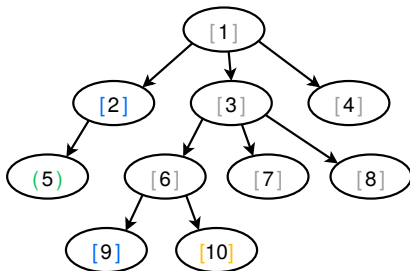
Breadth-first search (BFS): to\_visit is a queue

# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4, 3, 8, 7, 6, 10

BFS:



Depth-first search (DFS): to\_visit is a stack

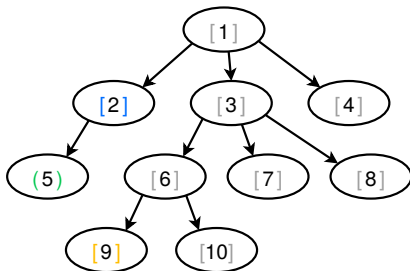
Breadth-first search (BFS): to\_visit is a queue

# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4, 3, 8, 7, 6, 10, 9

BFS:



Depth-first search (DFS): to\_visit is a stack

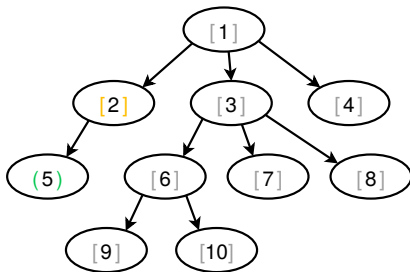
Breadth-first search (BFS): to\_visit is a queue

# Review: Ordering

In what order will BFS and DFS visit these states?  
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DFS:  
1, 4, 3, 8, 7, 6, 10, 9, 2

BFS:



Depth-first search (DFS): to\_visit is a stack

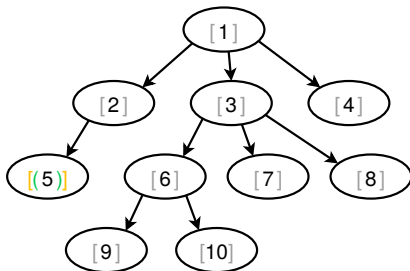
Breadth-first search (BFS): to\_visit is a queue

# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4, 3, 8, 7, 6, 10, 9, 2, 5

BFS:



Depth-first search (DFS): to\_visit is a stack

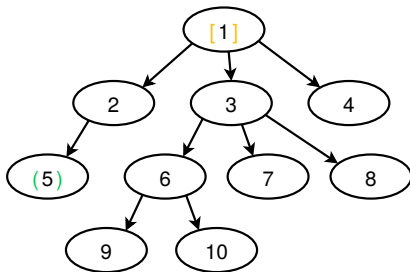
Breadth-first search (BFS): to\_visit is a queue

# Review: Ordering

In what order will BFS and DFS visit these states?  
(assuming they're added to to\_visit left-to-right)

DFS:  
1, 4, 3, 8, 7, 6, 10, 9, 2, 5

BFS:  
1



Depth-first search (DFS): to\_visit is a stack

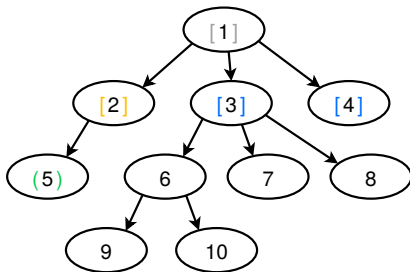
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# Review: Ordering

In what order will BFS and DFS visit these states?  
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DFS:  
1, 4, 3, 8, 7, 6, 10, 9, 2, 5

BFS:  
1, 2



Depth-first search (DFS): to\_visit is a stack

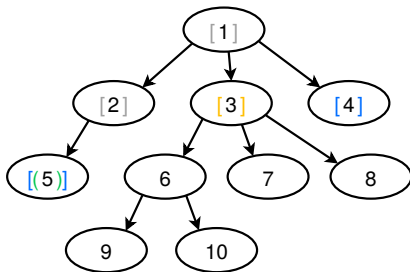
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# Review: Ordering

In what order will BFS and DFS visit these states?  
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DFS:  
1, 4, 3, 8, 7, 6, 10, 9, 2, 5

BFS:  
1, 2, 3



Depth-first search (DFS): to\_visit is a stack

Breadth-first search (BFS): to\_visit is a queue

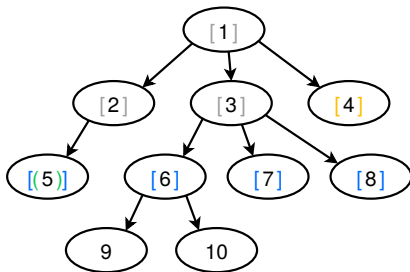


# Review: Ordering

In what order will BFS and DFS visit these states?  
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DFS:  
1, 4, 3, 8, 7, 6, 10, 9, 2, 5

BFS:  
1, 2, 3, 4



Depth-first search (DFS): to\_visit is a stack

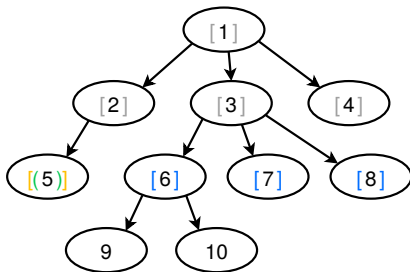
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In what order will BFS and DFS visit these states?  
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DFS:  
1, 4, 3, 8, 7, 6, 10, 9, 2, 5

BFS:  
1, 2, 3, 4 5



Depth-first search (DFS): to\_visit is a stack

Breadth-first search (BFS): to\_visit is a queue

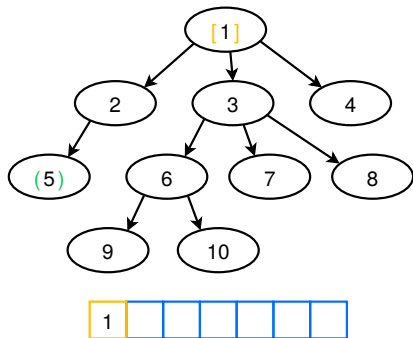
# Search Implementations

- ▶ Add the start state to to\_visit.
- ▶ Repeat:
  - ▶ Take a state off the to\_visit list.
  - ▶ If it's the goal state:
    - ▶ We're done!
  - ▶ If not:
    - ▶ Add all successor states to to\_visit.

```
def dfs(start_state):  
    s = Stack()  
    return search(start_state, s)  
  
def bfs(start_state):  
    q = Queue()  
    return search(start_state, q)  
  
def search(start_state, to_visit):  
    to_visit.add(start_state)  
  
    while not to_visit.is_empty():  
        current = to_visit.remove()  
  
        if current.is_goal():  
            return current  
        else:  
            for s in current.next_states():  
                to_visit.add(s)  
  
    return None
```

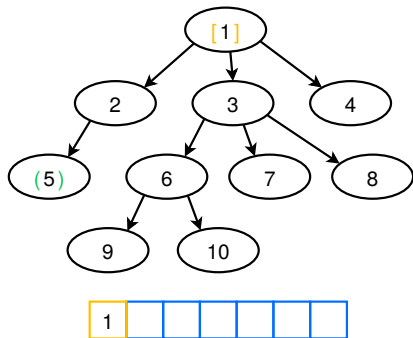
# Search Implementations

```
def search(state):  
    if state.is_goal():  
        return state  
    else:  
        for s in state.next_states():  
            result = search(s)  
            if result != None:  
                return result  
  
    return None
```



# Search Implementations

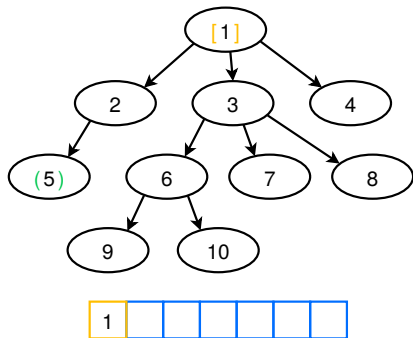
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Ordering?

# Search Implementations

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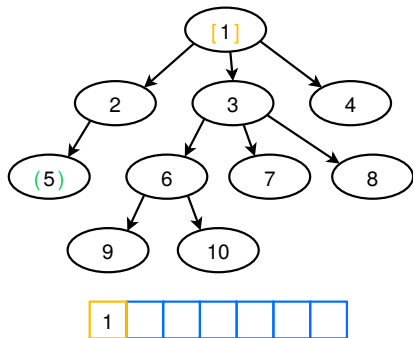


Ordering?

1, 2, 5

# Search Implementations

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Ordering?

1, 2, 5

What algorithm is this?

# Search Implementations

```
def search(state):  
    if state.is_goal():  
        return state  
    else:  
        for s in state.next_states():  
            result = search(s)  
            if result != None:  
                return result  
  
    return None
```

```
def search(state):  
    if state.is_goal():  
        return [state]  
    else:  
        result = []  
  
        for s in state.next_states():  
            result += search(s)  
  
    return result
```

What is the difference?



# Search Implementations

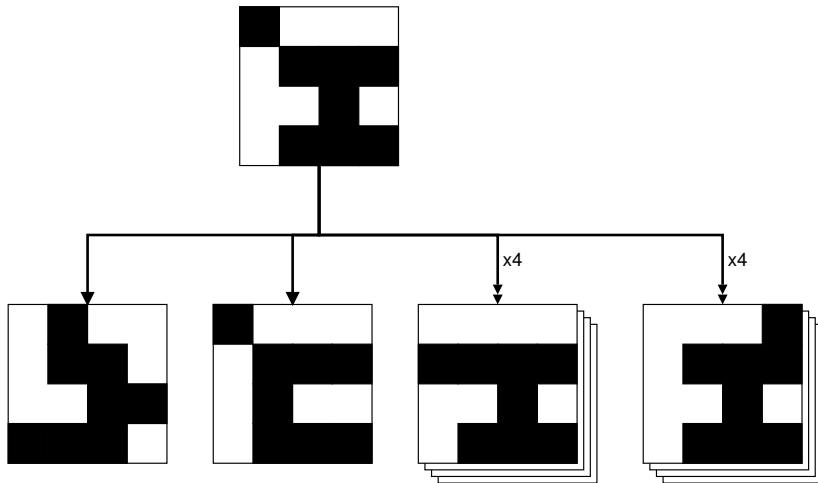
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def search(state):  
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    else:  
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    return result
```

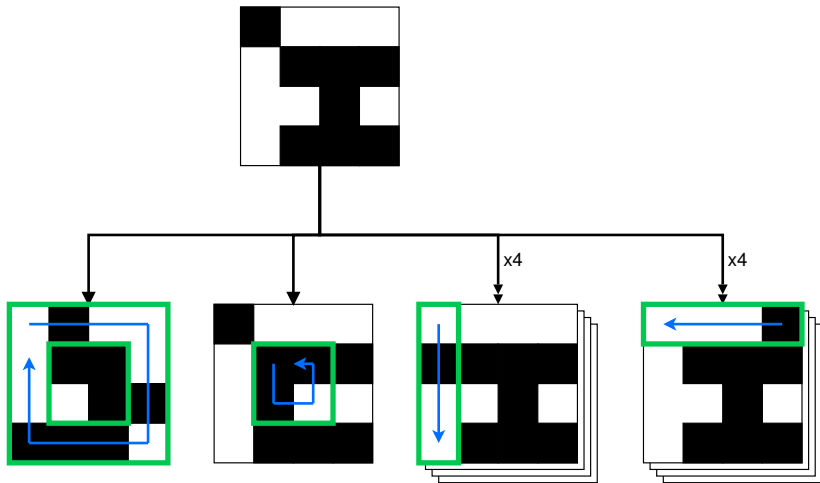
What is the difference?

Returns **all** solutions, not just one.

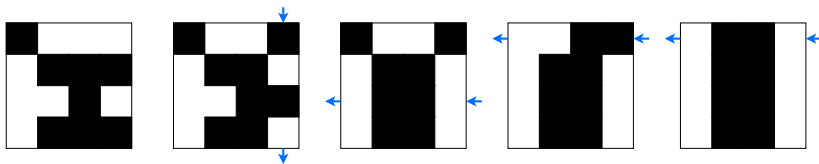
# Square Puzzle



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# Square Puzzle



# Square Puzzle

- ▶ How can we represent a state?
- ▶ How do we know if we're at a solution?
- ▶ How many next states does each state have?
- ▶ How can we get the next states?

# Square Puzzle

- ▶ How can we represent a state?

# Square Puzzle

- ▶ How can we represent a state?

```
( (0, 1, 1, 1),  
  (1, 0, 0, 0),  
  (1, 1, 0, 1),  
  (1, 0, 0, 0) )
```

# Square Puzzle

- ▶ How do we know if we're at a solution?



# Square Puzzle

- ▶ How do we know if we're at a solution?

```
def is_vert_solution(state):  
    for x in range(len(state)):  
        for y in range(len(state[0])):  
            first = state[x][0]  
            if state[x][y] != first:  
                return False  
    return True
```

```
def is_horiz_solution(state):  
    for y in range(len(state[0])):  
        for x in range(len(state)):  
            first = state[0][y]  
            if state[x][y] != first:  
                return False  
    return True
```

# Square Puzzle

- ▶ How many next states does each state have?
- ▶ How can we get the next states?

# Square Puzzle

- ▶ How many next states does each state have?
- ▶ How can we get the next states?

```
def swizzle(state):  
    lstate = as_list(state)  
    save = lstate[1][1]  
    lstate[1][1] = lstate[1][2]  
    lstate[1][2] = lstate[2][2]  
    lstate[2][2] = lstate[2][1]  
    lstate[2][1] = save  
    return as_tuple(lstate)
```

...

```
def next_states(state):  
    return [ pull_column(state, x) for x in range(len(state)) ]  
        + [ pull_row(state, y) for y in range(len(state[0])) ]  
        + [ swizzle(state), swizzle(state) ]
```

# Demo

119

search mode: **depth**  
memory: **on**  
colors: **2**

120

tempo: **medium**

121

visited: **124**  
to\_visit: **1215**  
path length: **<unknown>**

122

q: **quit**  
r: **reset search**  
R: **randomize puzzle**  
x: **toggle search mode**  
m: **toggle memory**  
c: **toggle complexity**

space: **step once**  
enter: **pause/unpause**  
t: **change tempo**

a: **auto-cursor**  
z: **reset cursor**  
←↑↓→: **move cursor**

search demo

# Search With Memory

```
def search(state):
    if state.is_goal():
        return [state]
    else:
        result = []

        for s in state.next_states():
            result += search(s)

    return result
```

```
def search(state, visited):
    # remember this state
    visited[state] = True

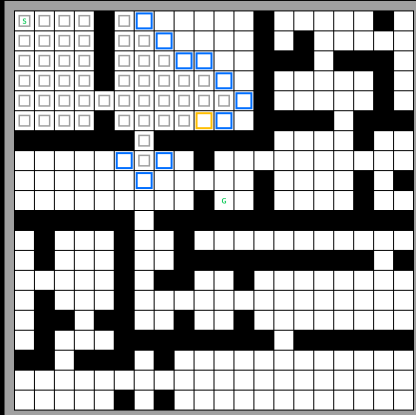
    if state.is_goal():
        return [state]
    else:
        result = []

        for s in state.next_states():
            # check if it's already visited
            if not(s in visited):
                result += search(s, visited)

    return result
```

# More Demo

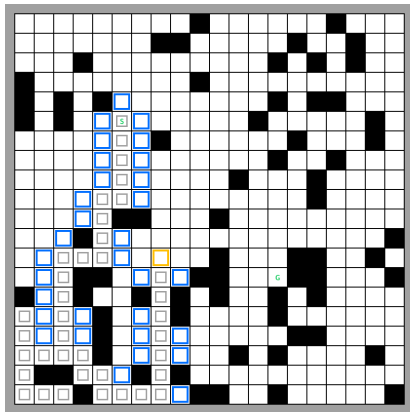
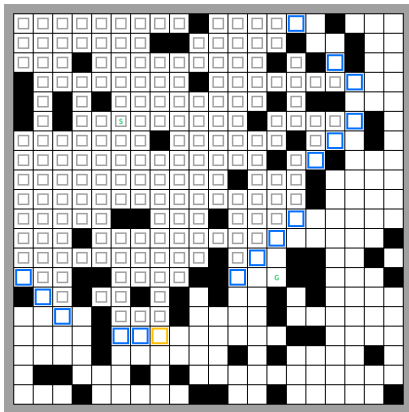
map type: **rooms**  
mode: **breadth**  
condition: **none**  
  
tempo: **fast**  
draw paths: **no**  
  
visited: **48**  
to\_visit: **11**  
path length: **<unknown>**



r: reset map  
R: switch map type  
l: change scale  
m: toggle search mode  
M: special search mode  
v: cycle conditions  
  
z: reset search  
space: step once  
enter: pause/unpause  
  
t: change tempo  
p: toggle path drawing  
v: cycle conditions  
  
click: toggle wall at cursor  
right-click: erase at cursor  
s: place start at cursor  
g: place goal at cursor  
1-4: place key 1-4  
ctrl-1-4: place lock 1-4

search demo

# Breadth or Depth?



# Breadth or Depth?

- ▶ What is the best case?
- ▶ What is the worst case?
- ▶ Time? Memory?



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- ▶ What is the best case?
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- ▶ How do these depend on the search space?

# Breadth or Depth?

- ▶ What is the best case?
- ▶ What is the worst case?
- ▶ Time? Memory?
- ▶ How do these depend on the search space?
- ▶ Quality of solutions?

# Breadth or Depth?

For a solution at depth  $d$  in a space with branching factor  $B$  and max depth  $M$ :

## BFS

- ▶ Best case:
  - ▶ Consider  $\sim B^d$  nodes
  - ▶ Remember  $\sim B^d$  nodes
- ▶ Worst case:
  - ▶ Same as the best case
- ▶ Features:
  - ▶ Consistent (but expensive)
  - ▶ Finds shortest paths

## DFS

- ▶ Best case:
  - ▶ Consider  $d$  nodes
  - ▶ Remember  $d$  nodes
- ▶ Worst case:
  - ▶ Consider  $\sim B^M$  nodes
  - ▶ Remember  $\sim B^M$  nodes
- ▶ Features:
  - ▶ Inconsistent
  - ▶ Can save memory if there aren't cycles

# Something More?

