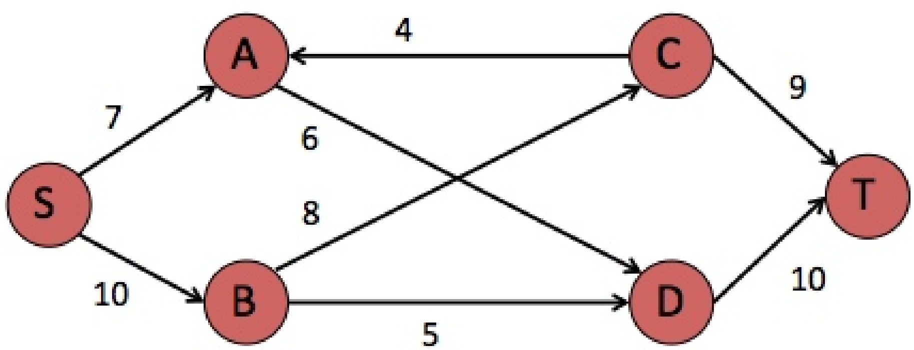
CS302 - Assignment 18

Due: Tuesday, April 30 at the beginning of class
Hand-in method: paper

1. **[13 points]** Go with the flow



(a) **[2 points]** Find the maximum flow f for the graph above and a minimum cut. Don't just state the max-flow value, but annotate the graph with the flow along each edge.

(b) **[2 points]** Draw the residual graph G_F at this maximum flow.

(c) **[2 points]** An edge of a network is called a *bottleneck edge* if increasing its capacity results in an increase in the maximum flow. List all of the bottleneck edges in the above network.

(d) **[2 points]** Give a simple example (containing at most four nodes) of a valid flow network which has no bottleneck edges.

(e) **[5 points]** Describe clearly (or write pseudocode for) an efficient algorithm to identify all the bottleneck edges in a network. *Hint:* It may be useful to calculate the max-flow first. State your running time.

2. **[4 points]** Suppose someone gives you a solution to a max-flow problem on some network (you can assume whatever form is convenient for how the solution is represented). Describe an efficient algorithm to determine whether the solution is indeed a maximum flow solution. State your running time. You will be graded on efficiency.

3. **[4 points]** Determine wether the following statement is true or false. If false, give a counterexample. If true, give a brief (but concrete) explanation justifying the statement.

   Given a flow network G, let (L, R) be a minimum capacity cut in the flow graph. If we increase the capacity of all of the edges in the graph by 1, then (L, R) is still a minimum capacity cut in this new graph.

4. **[4 points]** 26.1-3 (pg. 713). Show = prove :)