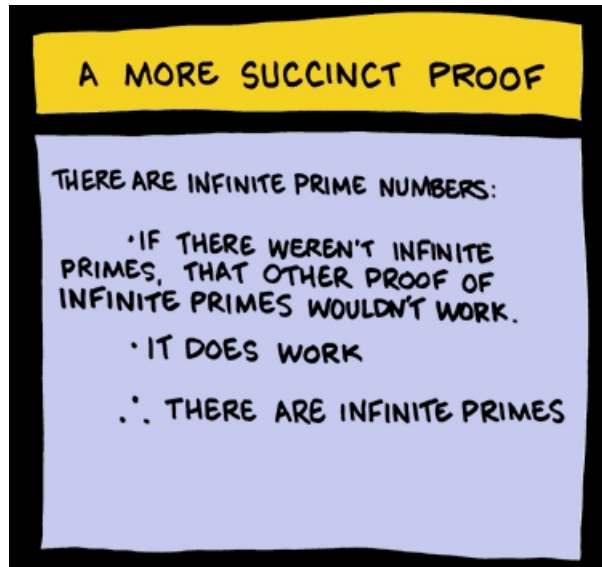# CS302 - Assignment 4

Due: Tuesday, Feb. 28 at the beginning of class
Hand-in method: paper

For this assignment you must use latex to generate your work.

1. [**5 points**] Algorithm A has a running time described by the recurrence $T(n) = 7T(n/2) + n^2$. A competing algorithm B has a running time described by the recurrence $T(n) = aT(n/4) + n^2$. What is the largest integer value for a such that B is asymptotically faster than A? Explain your answer. (Hint: use the master method.)

2. [**23 points**] There are some situations where we are asked to sort data that is *almost* sorted. A k-sorted array contains no element that

is more than k positions from it's position in the properly sorted array. For the questions below, A is a k-sorted array with $k << n$:

(a) (9 points) What are the runtimes of INSERTION-SORT, MERGE-SORT and QUICKSORT on A? Explain your answers.

(b) (5 points) What is the runtime of BUBBLE-SORT on A (see pseudocode below)? Explain your answer.

BUBBLE-SORT(A)

```
1   sorted ← false
2   while sorted = false
3              sorted ← true
4              for i ← 1 to length[A] −1
5                      if A[i] > A[i + 1]
6                              swap A[i] and A[i + 1]
7                              sorted ← false
```

(c) (10 points) Write pseudocode and provide the runtime for an algorithm that performs better than the above algorithms for sorting a k-sorted array.

(d) (5 points) Prove the correctness of your algorithm.

3. [**4 points**] A stable sorting algorithm is an algorithm where elements with equal value appear in the output (sorted) array in the same order as they do in the input array. Which of INSERTION-SORT, MERGE-SORT, BUBBLE-SORT and QUICKSORT are stable? Briefly explain your answers.

Just for fun

(1 brownie point) Give pseudocode for an algorithm that runs in time $O(n)$ and memory $O(1)$ that solves the following problem: Given an array A containing all integers $0, 1, ..., n$ except one, output the missing integer.