CS302 - Assignment 16

Due: Tuesday, April 24 at the beginning of class
Hand-in method: paper

Notes:

- Many of the algorithms below can be accomplished by either modifying the graph and applying a known algorithm or slightly modifying a known algorithm. Try thinking of these *first* as they will save you a lot of work, and writing :)
- You will be graded on efficiency!
- If not specified in the problem, you may assume whatever graph representation makes your algorithm more efficient (adjacency list or adjacency matrix). State which one you are using.

1. **[5 points]** Someone suggests to you the following algorithm for finding the shortest distance (sum of edge weights) path from node s to node t in a directed graph with some negative edges: add a large constant to each edge weight so that all the weights become positive, then run Dijkstra's algorithm starting at node s, and return the shorted path found to node t.

   Is this a valid method? Argue that it works correctly or give a counterexample.

2. **[8 Points]** Assume that you are given a graph and a source vertex. You are told that the graph has the following property: for every vertex v, the weights of the edges along the

shortest path from s to v increase monotonically. Describe an efficient algorithm to solve the single source shortest path problem for such a graph. Explain why your algorithm is correct and give the runtime.

You may assume that there are no negative weight cycles, though there may be negative edge weights.

3. **[8 points]** Given a directed graph $G = (V, E)$ with positive edge weights and a particular node $v_i \in V$, give an efficient algorithm for finding the shortest paths between **all pairs of nodes**, with the one restriction that these paths must all pass through v_i. Give the runtime of your algorithm. Points will be deducted for an inefficient algorithm.

4. **[8 points]** Given an undirected graph G with nonnegative edge weights $w_e \geq 0$. Suppose you have calculated the minimum spanning tree of G and also the shortest paths to all nodes from a particular node $s \in V$. Now, suppose that each edge weight is increased by 1, i.e. the new weights $w'_e = w_e + 1$.

   (a) (4 points) Does the minimum spanning tree change? Give an example where it does or prove that it cannot change.

   (b) (4 points) Do the shortest paths from s change? Given an example where it does or prove that it cannot change.

5. **[12 points]** T/F. State whether the following are true or false **AND** give a brief, but compelling, justification of your answer. Assume graph G is undirected and connected.

   If you decide any of the answers are T, make sure that you think very thoroughly through your proof since it can be easy to miss corner cases.

   (a) (3 points) If graph G has more than $|V| - 1$ edges, and there is a unique largest edge, then this edge cannot be part of the MST.

   (b) (3 points) If G has a cycle with a unique heaviest edge e, then e cannot be part of the MST.

   (c) (3 points) If the lightest edge in G is unique, then it must be part of every MST.

   (d) (3 points) The shortest path between two nodes is necessarily part of some MST.