

# CS41B RECURSION

David Kauchak  
CS 52 – Fall 2015

## Admin

Midterm back on Thursday

**Assignments**

- Assignment 4: due Monday (10/12 at 11:59pm)
- Assignment 5: due Friday (10/23 at 5pm)
- Assignment 6: due Monday (11/2 at 11:59pm)

Survey in assignment 4

Academic Honesty

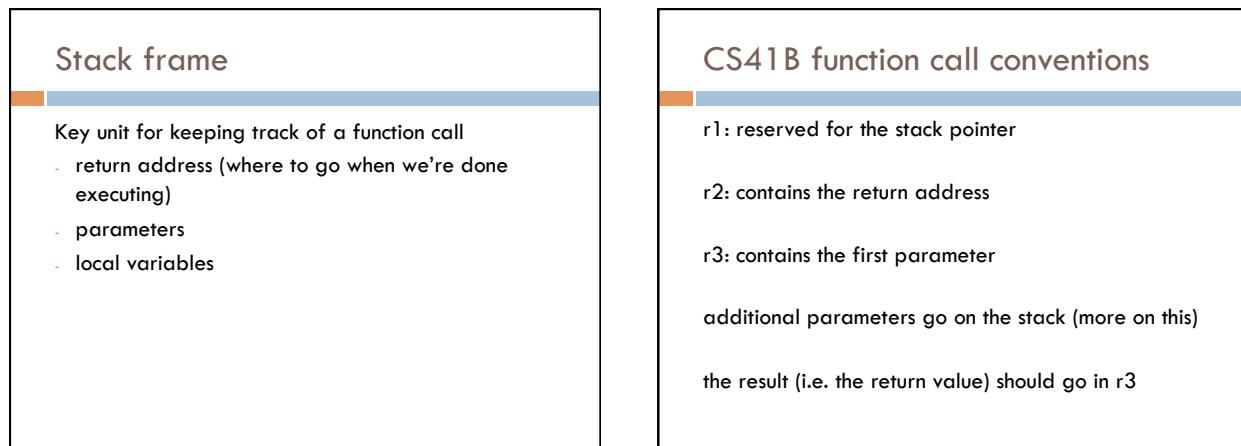
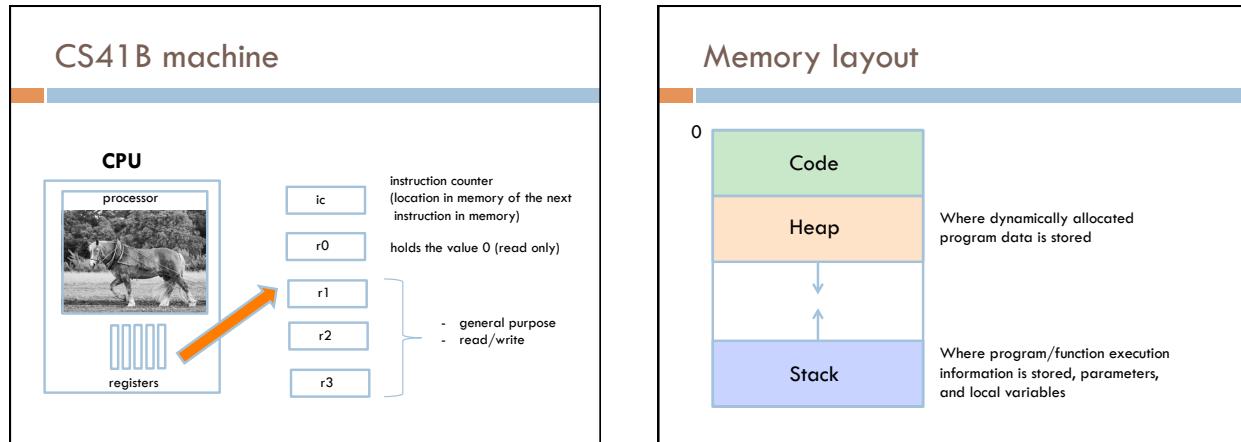
## Academic Honesty

A few rules to follow for this course to keep you out of trouble:

- If you talk with someone in the class about a problem, you should not take notes. If you understand the material you talked about, you should be able to recreate it on your own.
- Similarly, if you talk with someone, you must wait 5 minutes before resuming work on the problem. Stretch. Use the restroom. Go for a quick walk. This will ensure that you really understand the material.
- You may not sit next to (or where you can see the screen of) anyone you are talking with about the assignment.
- The only time you may look at someone else's screen is if they are asking you for help with a basic programming problem (e.g. syntax error). You should not look at someone else's code to help yourself!

## Examples from this lecture

<http://www.cs.pomona.edu/~dkauchak/classes/cs52/examples/cs41b/>



## Structure of a single parameter function

```

fname
    psh r2          ; save return address on stack
    ...
    ; do work using r3 as argument
    ; put result in r3
    pop r2          ; restore return address from stack
    jmp r2          ; return to caller

```

**conventions:**

- argument is in r3
- r1 is off-limits since it's used for the stack pointer
- return value goes in r3

## Our first function call

```

loa r3 r0          ; get variable
lcw r2 increment   ; call increment
cal r2 r2

sto r0 r3          ; write result,
hlt                ; and halt

increment
    psh r2          ; save the return address on the stack
    adc r3 r3 1      ; add 1 to the input parameter
    pop r2          ; get the return address from stack
    jmp r2          ; go back to where we were called from

```

## Functions with multiple arguments

```

fname
    psh r2          ; save return address on stack
    loa r2 r1 4      ; load the second parameter into r2
    ...
    ; do work using r3 and r2 as arguments
    ; put result in r3
    pop r2          ; restore return address from stack
    jmp r2          ; return to caller

```

**conventions:**

- first argument is in r3
- r1 is off-limits since it's used for the stack pointer
- return value goes in r3

## Functions with multiple arguments

```

fname
    psh r2          ; save return address on stack
    loa r2 r1 4      ; load the second parameter into r2
    ...
    ; do work using r3 and r2 as arguments
    ; put result in r3
    pop r2          ; restore return address from stack
    jmp r2          ; return to caller

```

1oa	RR[S]	dest = mem[src0 + arg]
-----	-------	------------------------

What does this operation do? What is the 4?

## Functions with multiple arguments

```

fname
psh r2          ; save return address on stack
loa r2 r1 4    ; load the second parameter into r2
...
; do work using r3 and r2 as arguments
; put result in r3
pop r2          ; restore return address from stack
jmp r2          ; return to caller

```

loa	RR[S]	dest = mem[src0 + arg]
-----	-------	------------------------

- r1 is the stack pointer and points at the top (next) slot
- stacks grow towards smaller memory values

## Functions with multiple arguments

```

fname
psh r2          ; save return address on stack
loa r2 r1 4    ; load the second parameter into r2
...
; do work using r3 and r2 as arguments
; put result in r3
pop r2          ; restore return address from stack
jmp r2          ; return to caller

```

loa	RR[S]	dest = mem[src0 + arg]
-----	-------	------------------------

- r1 is the stack pointer and points at the top (next) slot
- stacks grow towards smaller memory values
- r1+2 is then the top value of the stack
- r1+4 is the 2<sup>nd</sup> value of the stack

## Multiple arguments

```

max
psh r2
loa r2 r1 4

bge r3 r2 endif      What does this code do?
adc r3 r2 0
endif
pop r2
jmp r2

```

## Multiple arguments

```

max
psh r2
loa r2 r1 4

bge r3 r2 endif      max, as a function!
adc r3 r2 0
endif
pop r2
jmp r2

```

## Calling max

```

loa r3 r0
loa r2 r0
psh r2
lcw r2 max
cal r2 r2
pop r2
sto r0 r3
hlt

```

Anything different?

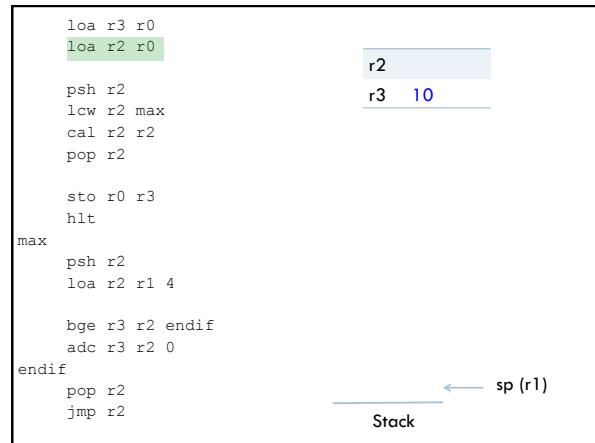
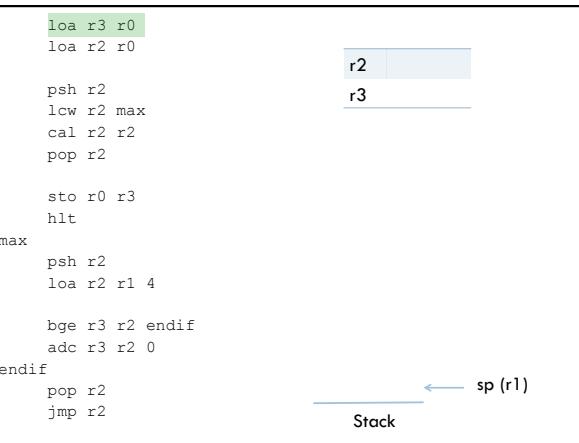
## Calling max

```

loa r3 r0
loa r2 r0
psh r2
lcw r2 max
cal r2 r2
pop r2
sto r0 r3
hlt

```

For the second argument,  
push it on the stack



```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2

    sto r0 r3
    hlt
max
    psh r2
    loa r2 r1 4

    bge r3 r2 endif
    adc r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

Stack

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2

    sto r0 r3
    hlt
max
    psh r2
    loa r2 r1 4

    bge r3 r2 endif
    adc r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

2

← sp (r1)

Stack

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2

    sto r0 r3
    hlt
max
    psh r2
    loa r2 r1 4

    bge r3 r2 endif
    adc r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

2

← sp (r1)

Stack

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2

    sto r0 r3
    hlt
max
    psh r2
    loa r2 r1 4

    bge r3 r2 endif
    adc r3 r2 0
endif
    pop r2
    jmp r2

```

Notice that we overwrote the value in r2

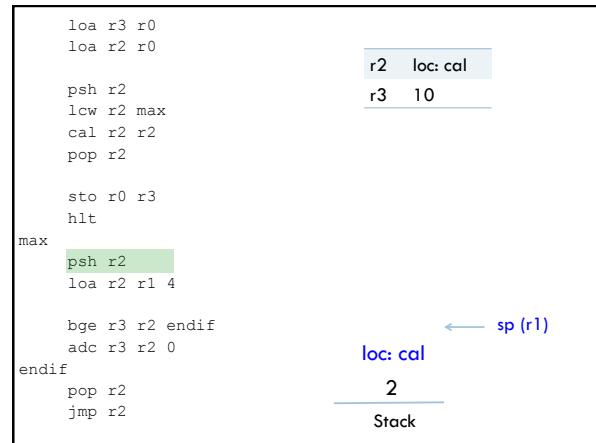
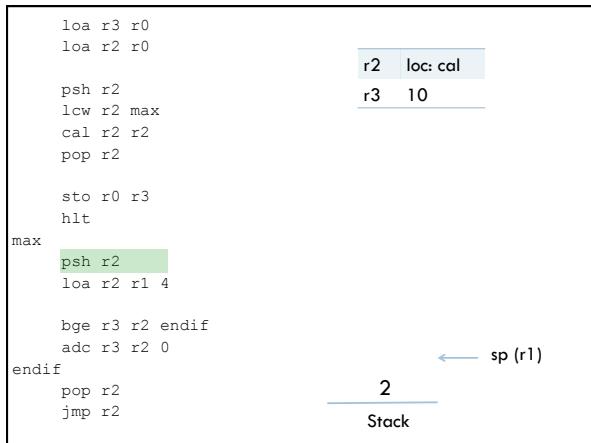
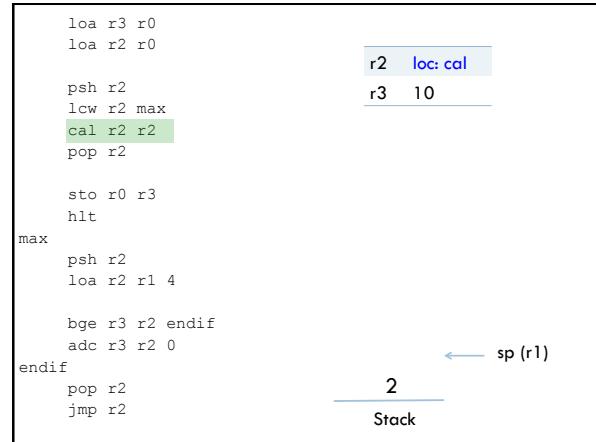
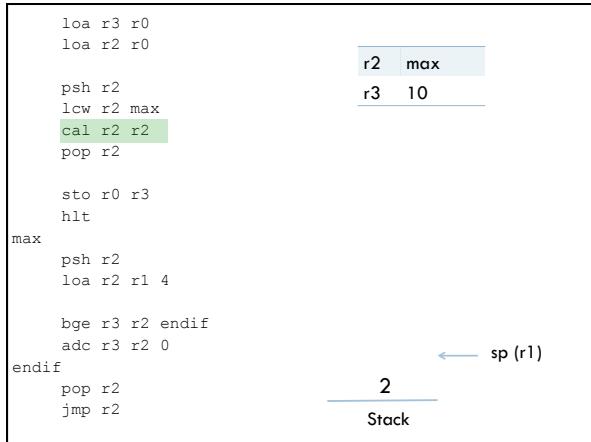
If we hadn't save it on the stack, it would have been lost

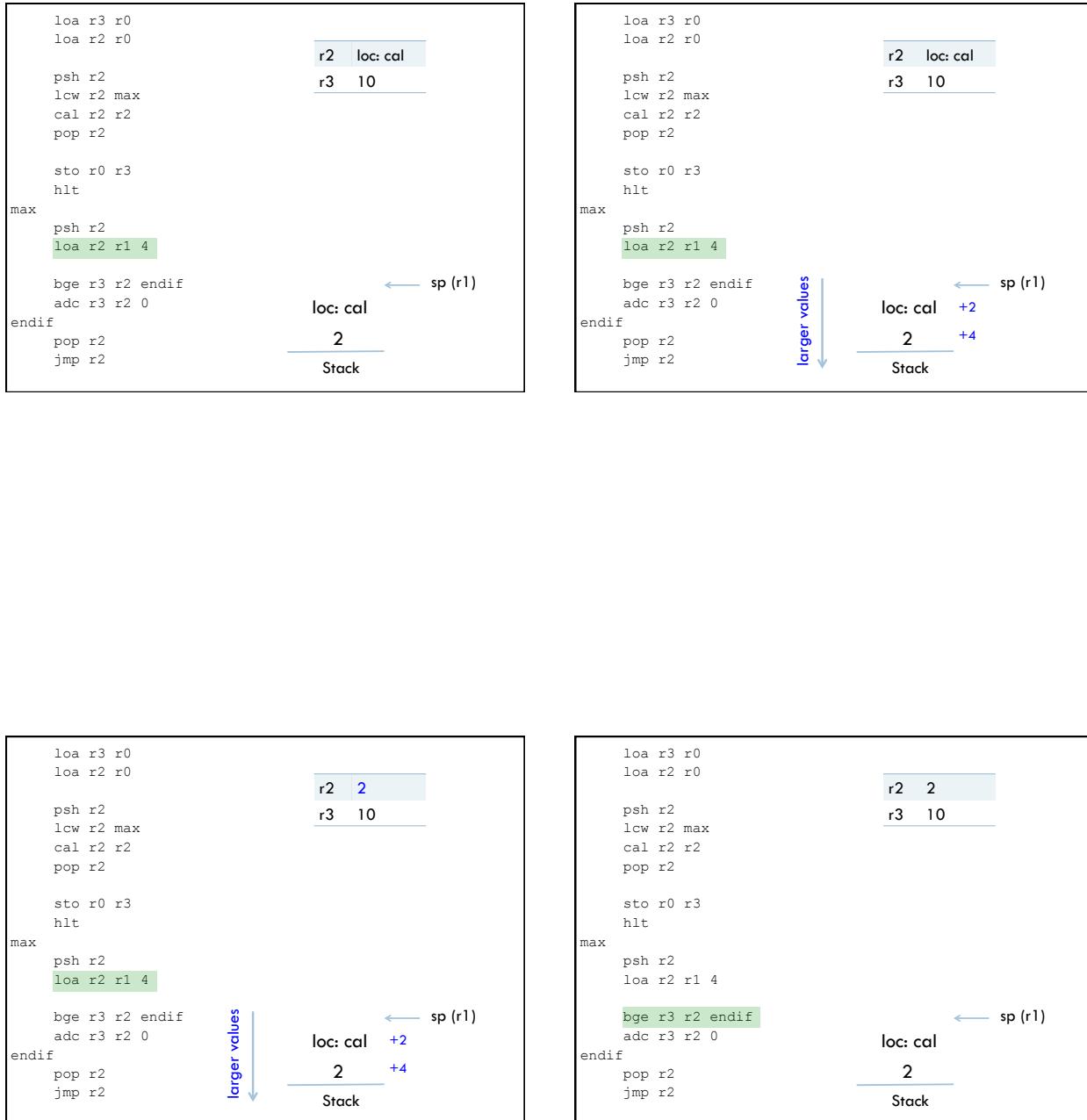
← sp (r1)

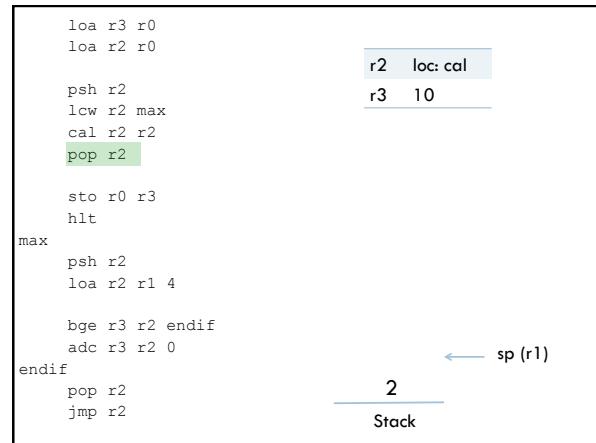
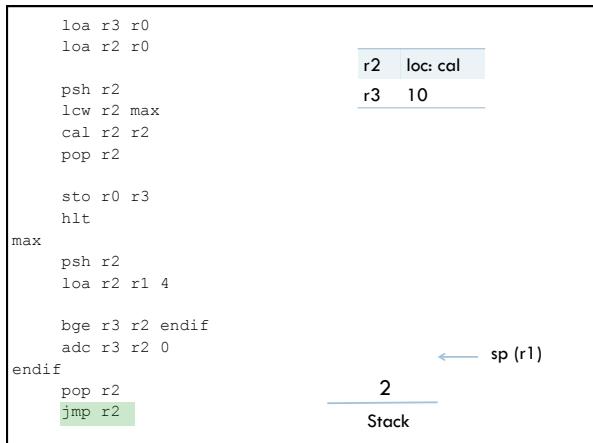
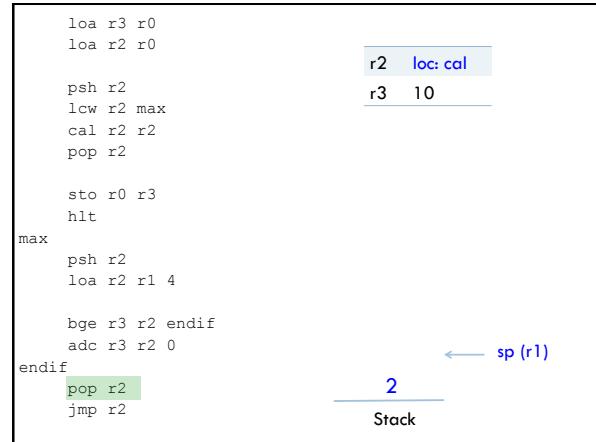
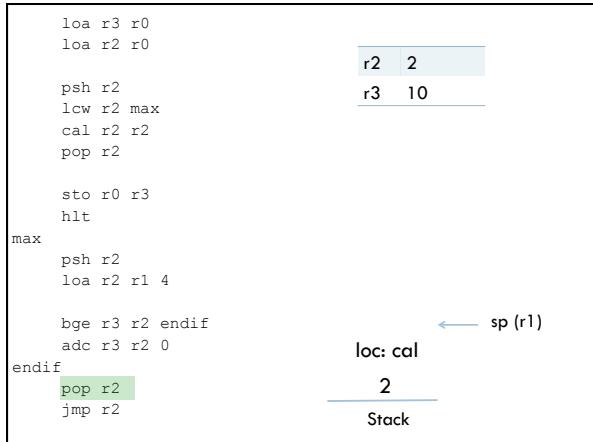
2

← sp (r1)

Stack







```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2
    sto r0 r3
    hlt
max
    psh r2
    loa r2 r1 4
    bge r3 r2 endif
    adc r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

Stack

r2	2
r3	10

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2
    sto r0 r3
    hlt
max
    psh r2
    loa r2 r1 4
    bge r3 r2 endif
    adc r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

Stack

r2	2
r3	10

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2
    sto r0 r3
    hlt
max
    psh r2
    loa r2 r1 4
    bge r3 r2 endif
    adc r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

Stack

r2	2
r3	10

10!

```

    loa r3 r0
    loa r2 r0
    psh r2
    lcw r2 max
    cal r2 r2
    pop r2
    sto r0 r3
    hlt
max
    psh r2
    loa r2 r1 4
    bge r3 r2 endif
    adc r3 r2 0
endif
    pop r2
    jmp r2

```

← sp (r1)

Stack

r2	2
r3	10

## Real structure of CS41B program

```

; great comments at the top!
;
lcw r1 stack           Save address of highest end
                        (highest address) of the stack in r1

instruction1      ; comment
instruction2      ; comment
...
hlt

;
; stack area: 50 words
;
dat 100
stack
end

```

} Reserve 50 words for the stack

## Recursion

```

int mystery(int a, int b){
    if( b <= 0 ){
        return 0
    } else
        return a + mystery(a, b-1)
}

```

What does this function do?

## Recursion

```

int mystery(int a, int b){
    if( b <= 0 ){
        return 0
    } else
        return a + mystery(a, b-1)
}

```

Multiplication...  $a \cdot b$  (assuming b is positive)

Note to future Dave from past Dave: write the function up on the board ☺

```

mult
psh r2      ; save the return address
psh r3      ; save first argument, a, on stack
loa r2 r1 6  ; get at the 2nd argument, b
              ; b = r2, a = r3
              }

bit r0 r2 else ; 0 < r2, i.e. recursive case
adc r3 r0 0   ; return 0
brs endif

else
sbc r2 r2 1  ; r2 = b-1
psh r2      ; add r2 as 2nd argument

lcw r2 mult  ; call mult recursively
cal r2 r2    ; pop 2nd argument off stack
pop r2      ; load a into r2 off of the stack
add r3 r3 r2 ; r3 = a + mult(a, b-1)
              answer calculation
endif

pop r0      ; remove first argument from stack
pop r2      ; get the return address
jmp r2      ; return

```

```

mult
    psh r2      ; save the return address
    psh r3      ; save first argument, a, on stack
    loa r2 r1 6 ; get at the 2nd argument, b
                  ; b = r2, a = r3

    blt r0 r2 else ; 0 < r2, i.e. recursive case
    cdc r3 r0 0   ; return 0
    brs endif

else
    sbc r2 r2 1 ; r2 = b-1
    psh r2      ; add r2 as 2nd argument, r3 shouldn't have changed

    lcw r2 mult ; call mult recursively
    cal r2 r2

    pop r2      ; pop 2nd argument off stack

    loa r2 r1 2 ; load a into r2 off of the stack
    add r3 r3 r2 ; r3 = a + mult(a, b-1)

endif
    pop r0      ; remove first argument from stack
    pop r2      ; get the return address
    imp r2      ; return

```

Notice symmetry of psh and pop

## Calling mult

```

loa r3 r0
loa r2 r0

```

```

psh r2
lcw r2 mult
cal r2 r2

```

```
pop r2
```

```

sto r0 r3
hlt

```

r2
r3

← sp (r1)

Stack

## Calling mult

```

loa r3 r0
loa r2 r0

```

r2
r3

```

psh r2
lcw r2 mult
cal r2 r2

```

```
pop r2
```

```

sto r0 r3
hlt

```

← sp (r1)

Stack

## Calling mult

```

loa r3 r0
loa r2 r0

```

r2
r3 6

```

psh r2
lcw r2 mult
cal r2 r2

```

```
pop r2
```

```

sto r0 r3
hlt

```

← sp (r1)

Stack

## Calling mult

```
loa r3 r0
loa r2 r0
```

```
psh r2
lcw r2 mult
cal r2 r2
```

```
pop r2
```

```
sto r0 r3
hlt
```

r2	
r3	6

← sp (r1)

Stack

## Calling mult

```
loa r3 r0
loa r2 r0
```

```
psh r2
lcw r2 mult
cal r2 r2
```

```
pop r2
```

```
sto r0 r3
hlt
```

r2	2
r3	6

← sp (r1)

Stack

## Calling mult

```
loa r3 r0
loa r2 r0
```

```
psh r2
lcw r2 mult
cal r2 r2
```

```
pop r2
```

```
sto r0 r3
hlt
```

r2	2
r3	6

← sp (r1)

Stack

## Calling mult

```
loa r3 r0
loa r2 r0
```

```
psh r2
lcw r2 mult
cal r2 r2
```

```
pop r2
```

```
sto r0 r3
hlt
```

r2	2
r3	6

← sp (r1)

Stack

## Calling mult

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r2

sto r0 r3
hlt

```

r2	2
r3	6

 $\leftarrow \text{sp (r1)}$ 2  
Stack

## Calling mult

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r2

sto r0 r3
hlt

```

r2	loc: mult
r3	6

 $\leftarrow \text{sp (r1)}$ 2  
Stack

## Calling mult

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r2

sto r0 r3
hlt

```

r2	loc: mult
r3	6

 $\leftarrow \text{sp (r1)}$ 2  
Stack

## mult

```

psh r2
psh r3
loa r2 r1 6

bit r0 r2 else
adc r3 r0 0
brs endif

else
    sbc r2 r2 1
    psh r2

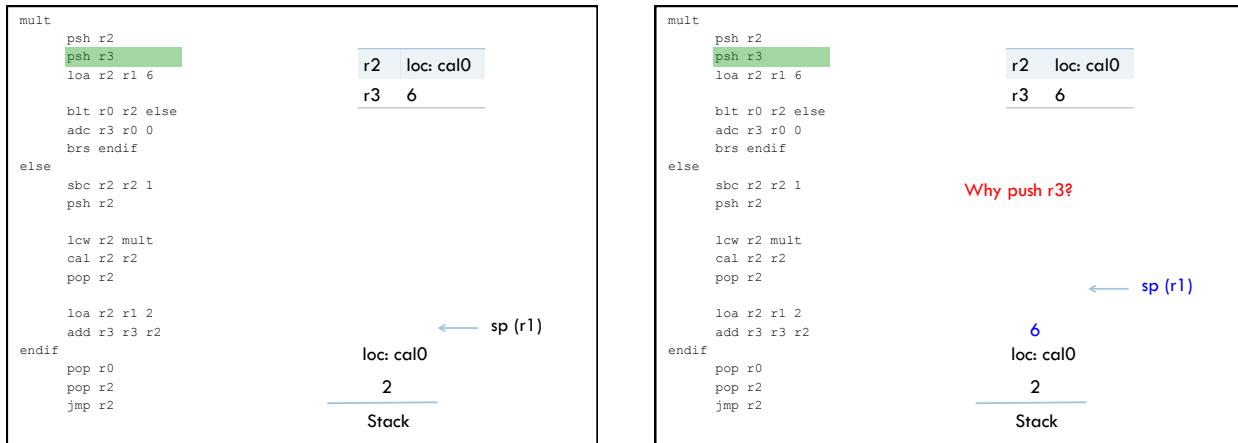
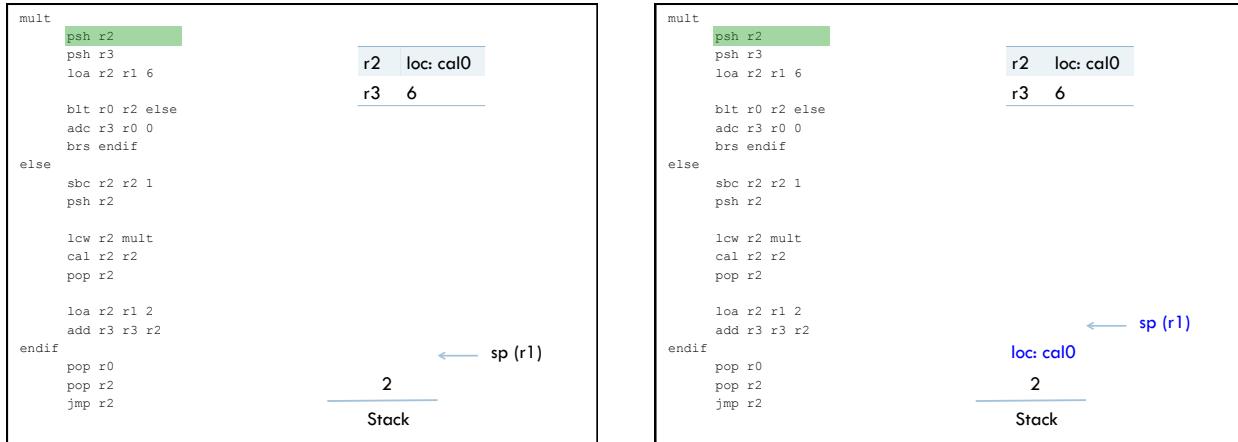
    lcw r2 mult
    cal r2 r2
    pop r2

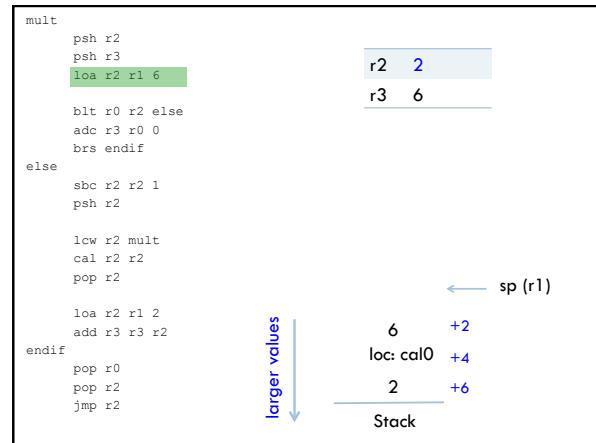
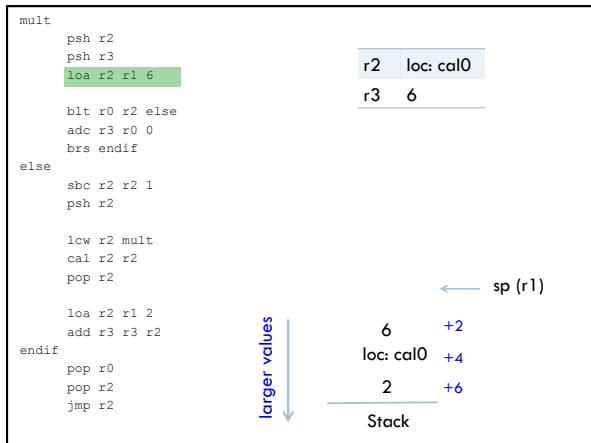
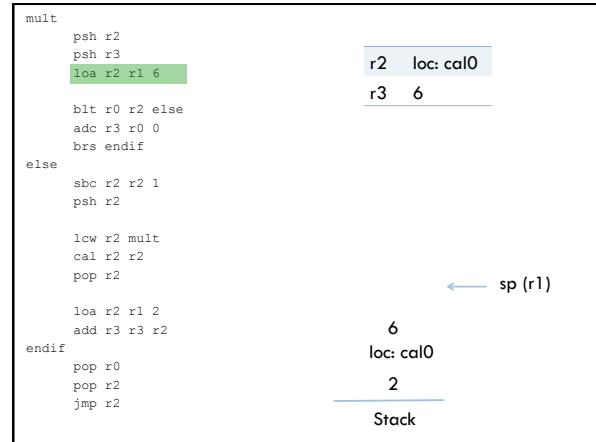
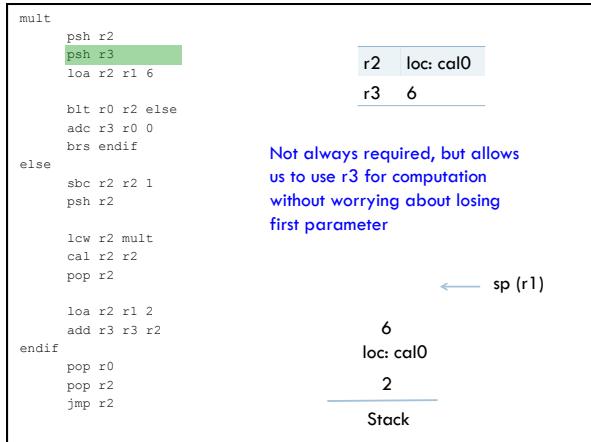
    loa r2 r1 2
    add r3 r3 r2
endif
    pop r0
    pop r2
    jmp r2

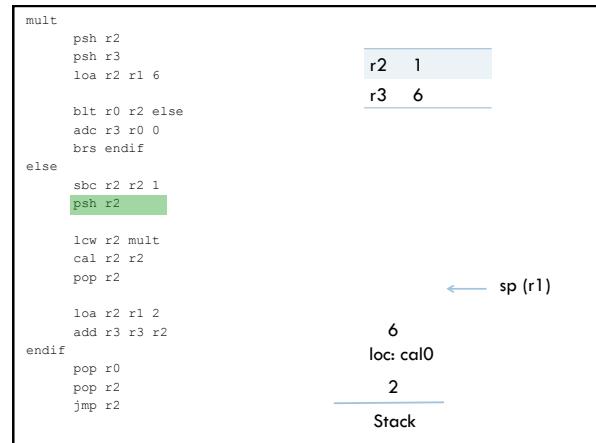
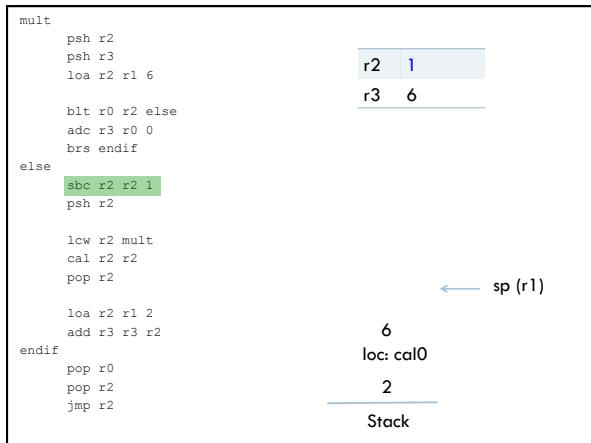
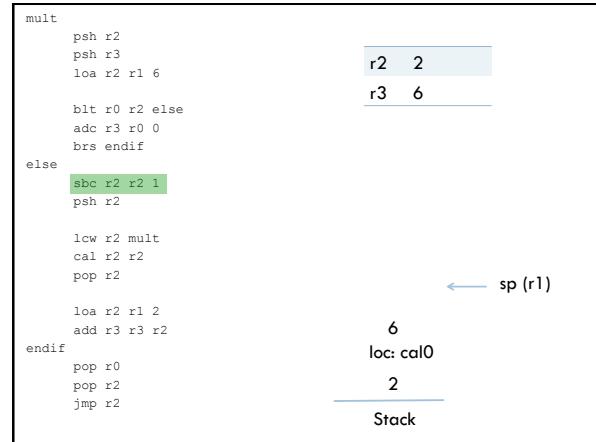
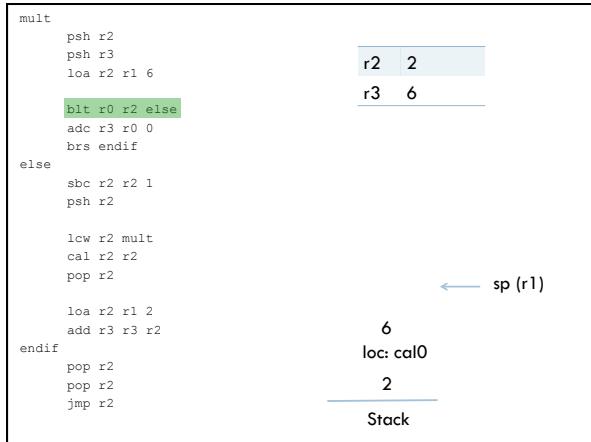
```

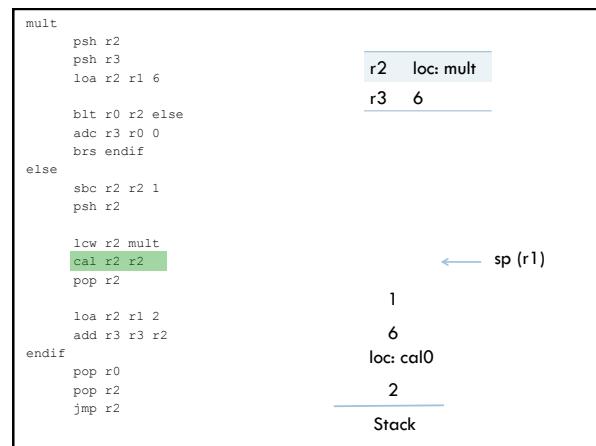
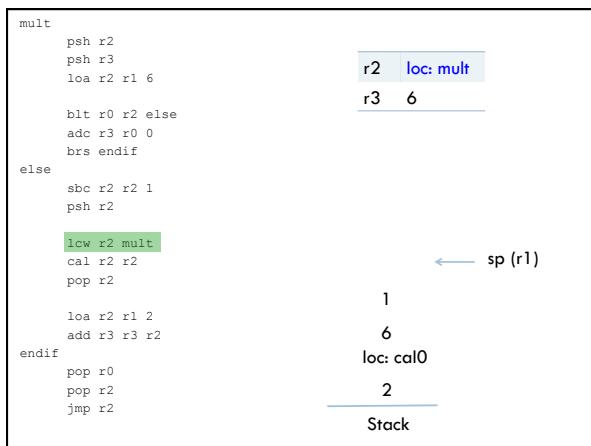
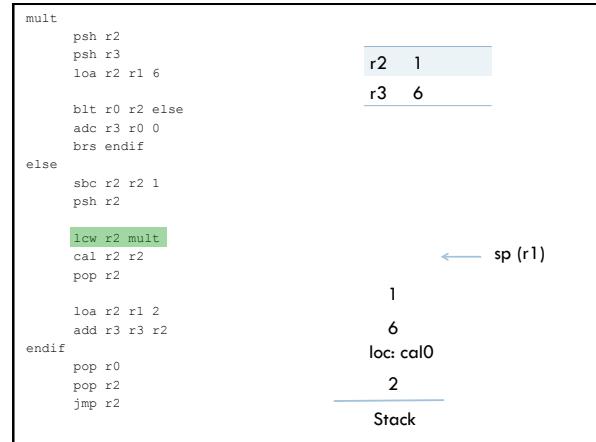
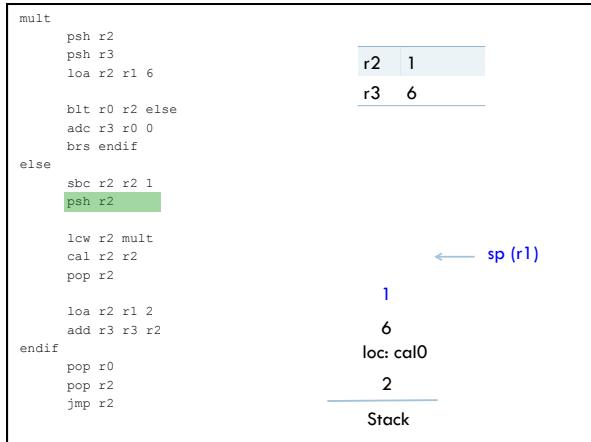
r2	loc: cal0
r3	6

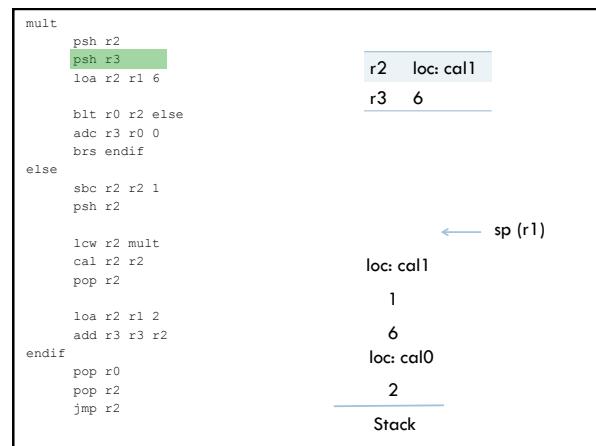
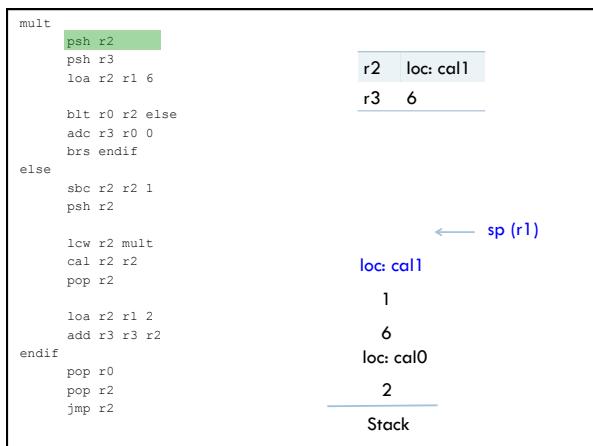
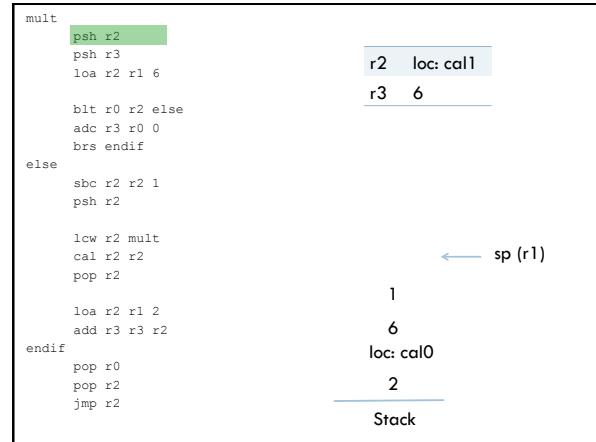
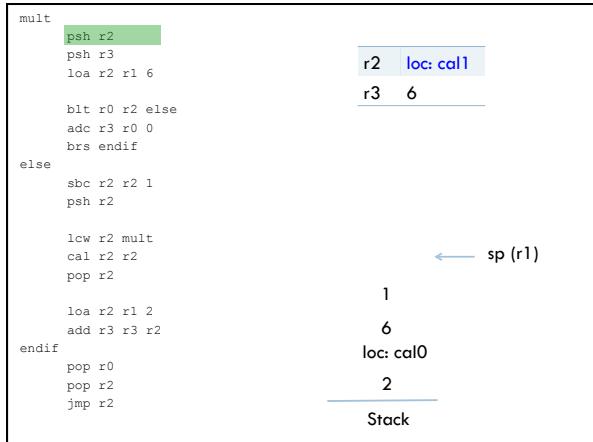
 $\leftarrow \text{sp (r1)}$ 2  
Stack

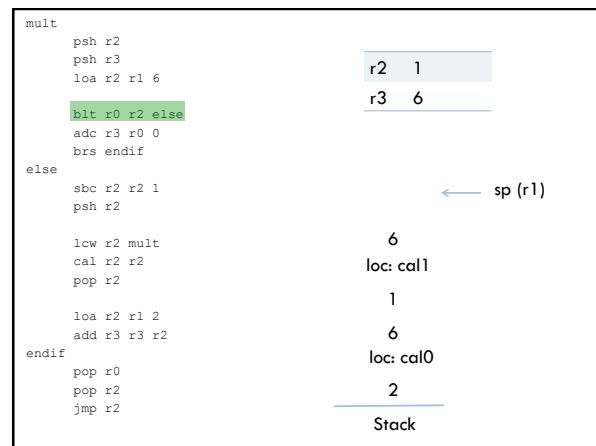
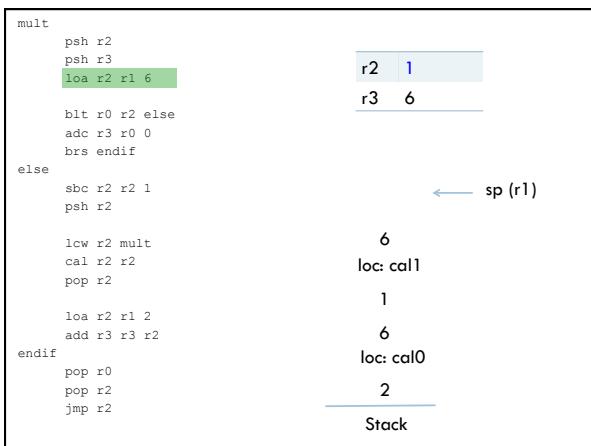
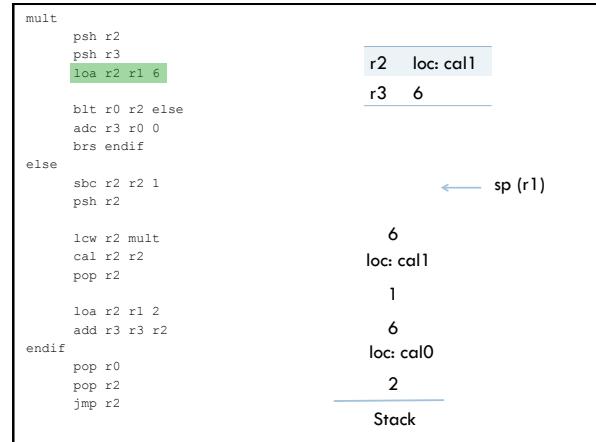
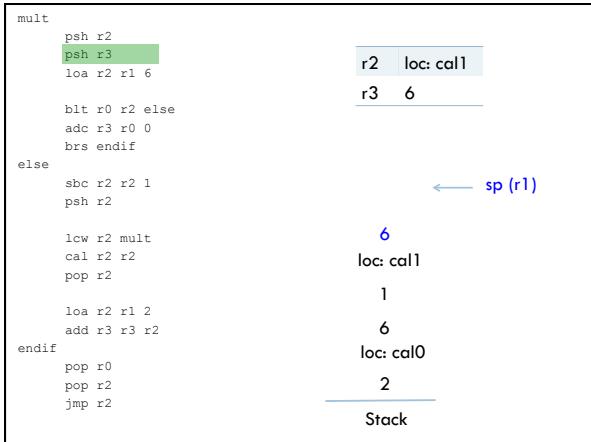


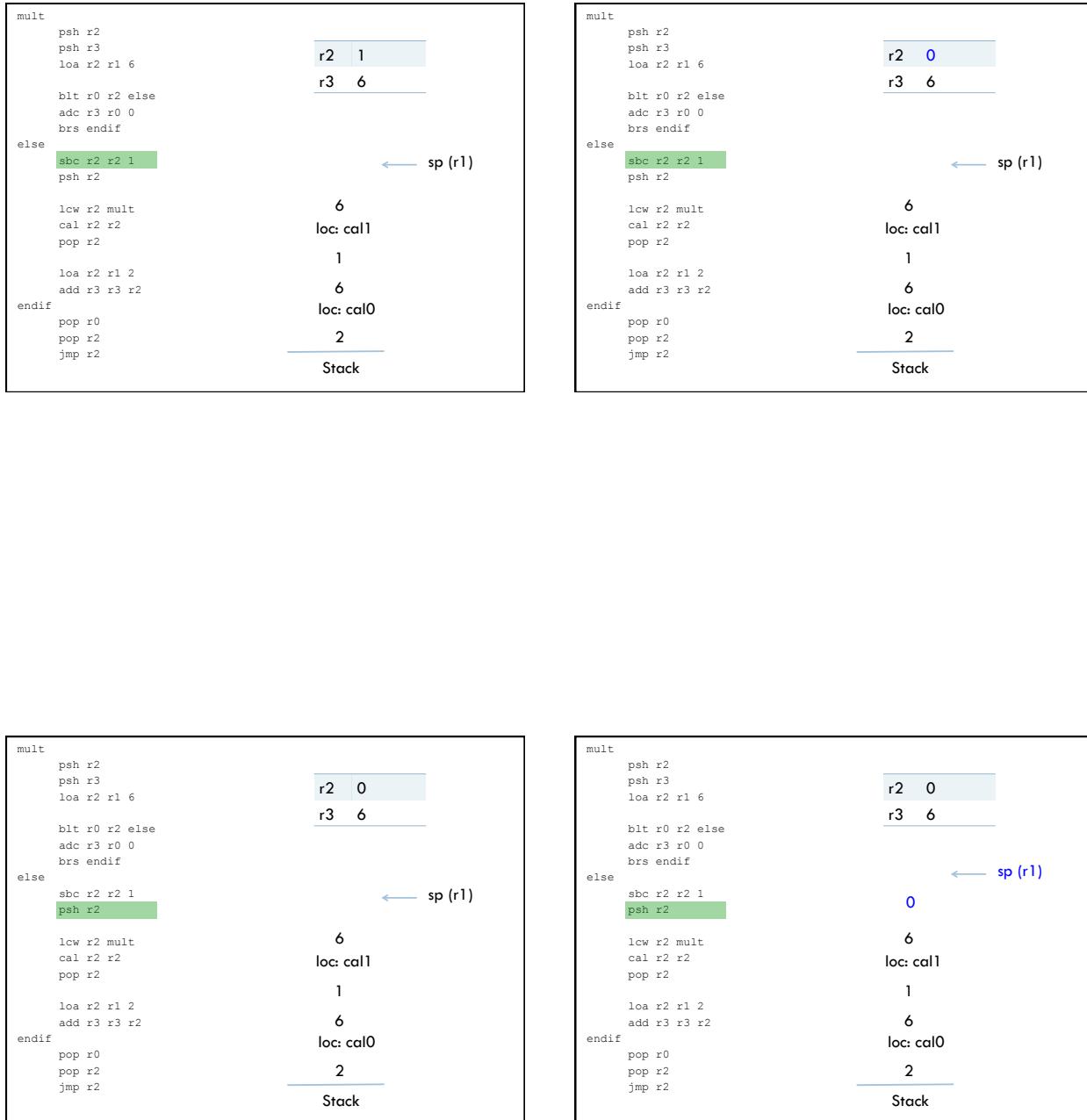


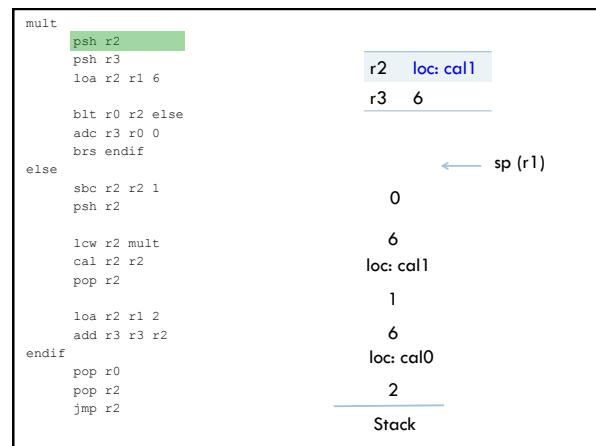
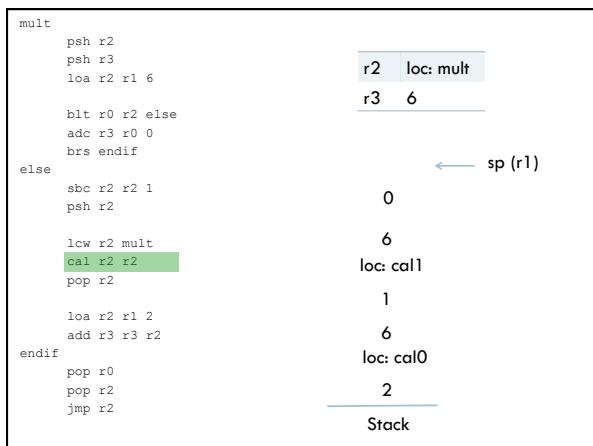
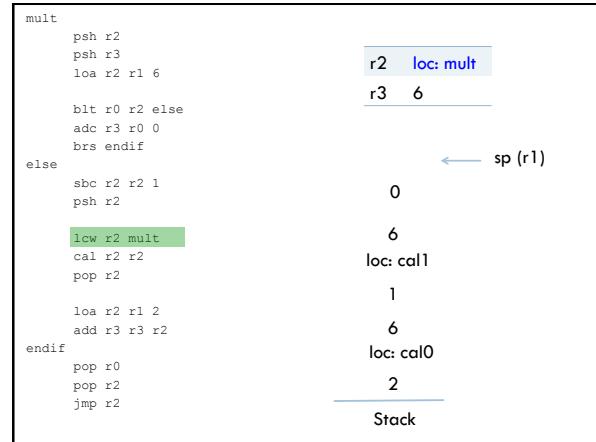
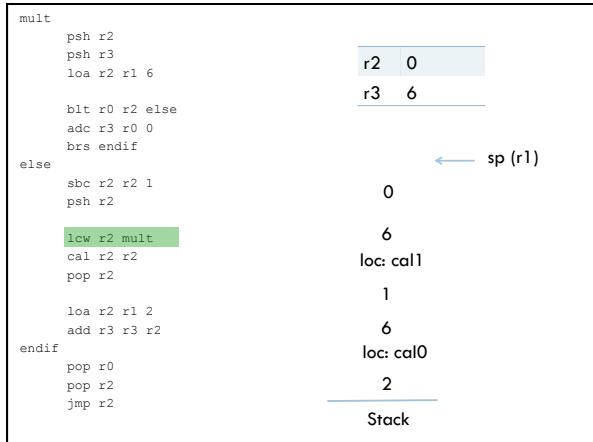


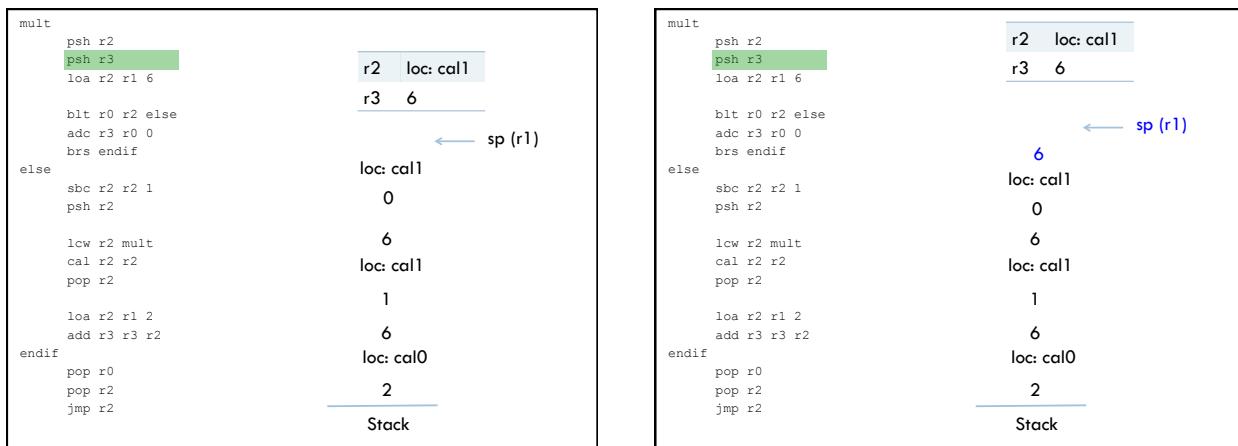
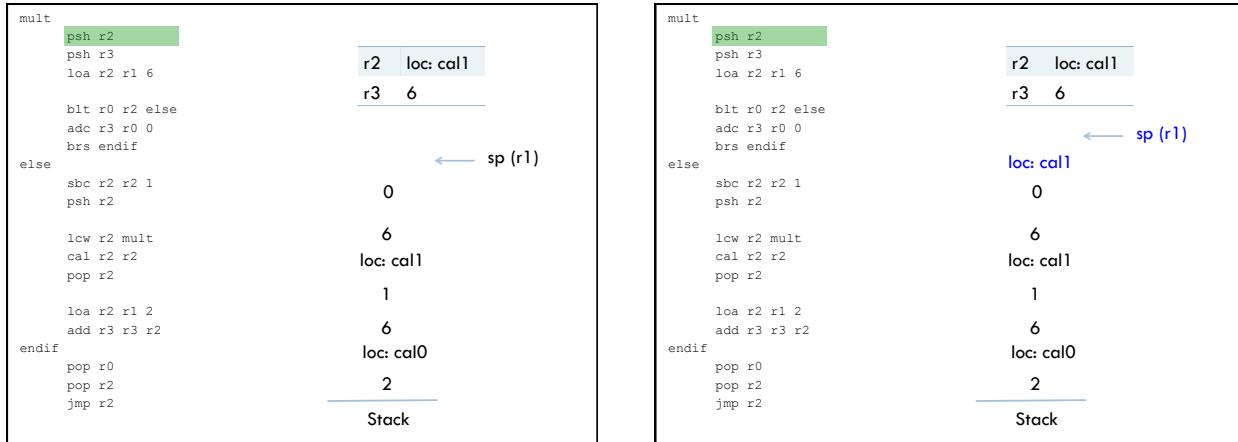


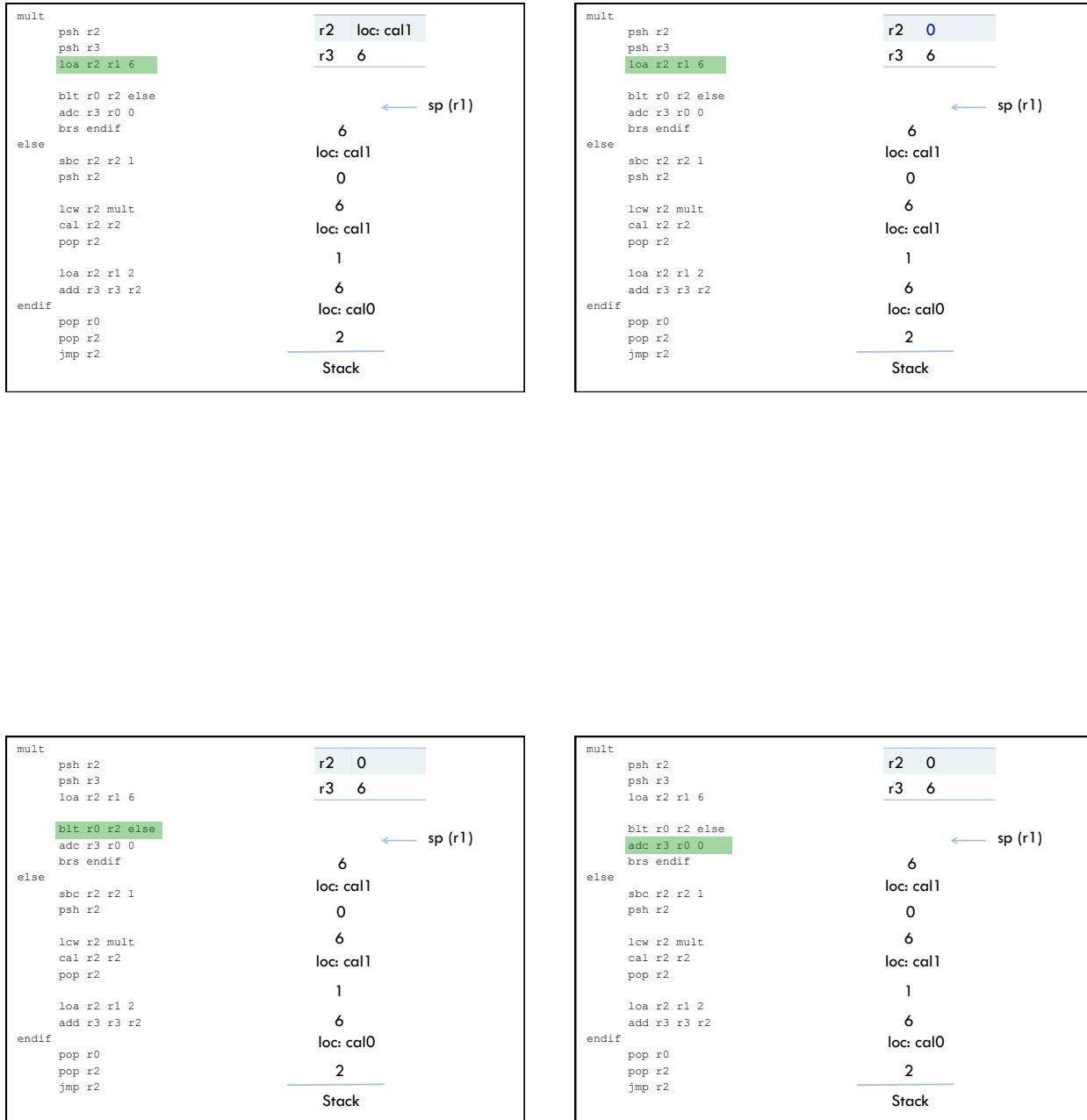


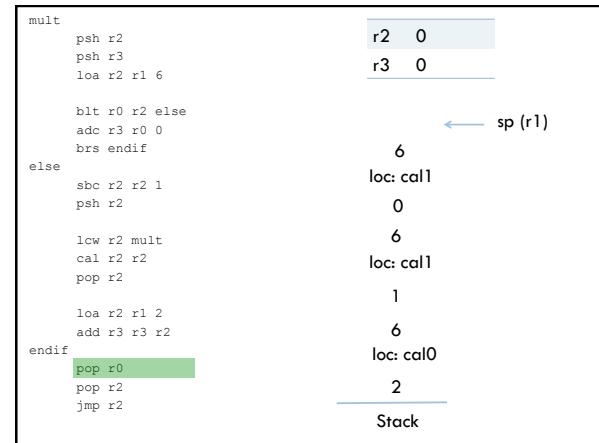
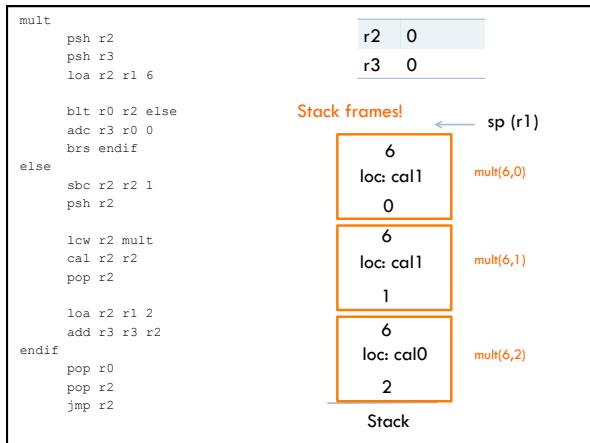
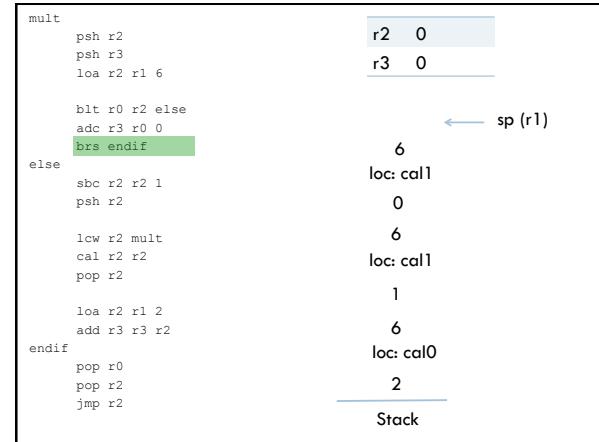
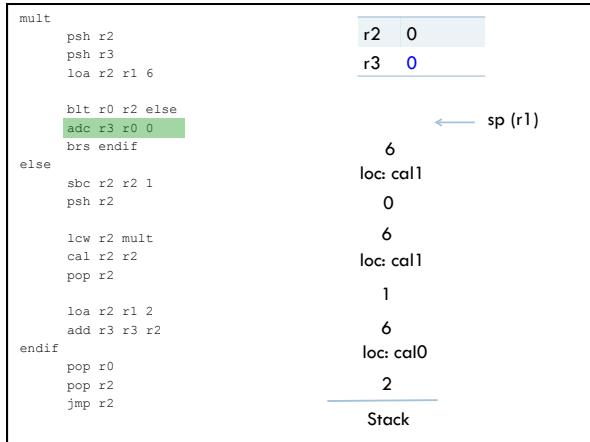












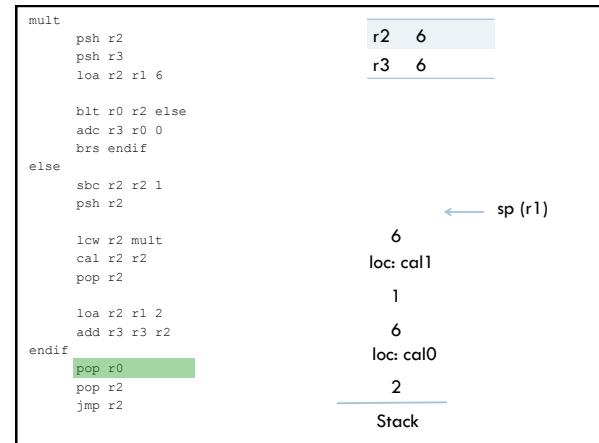
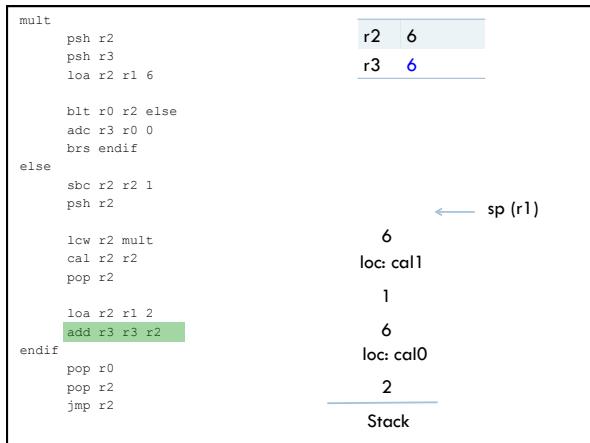
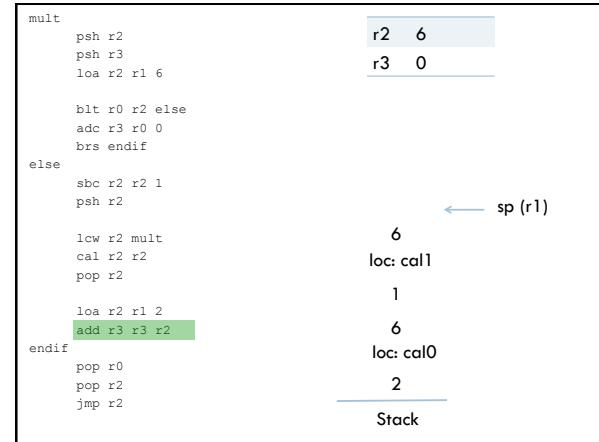
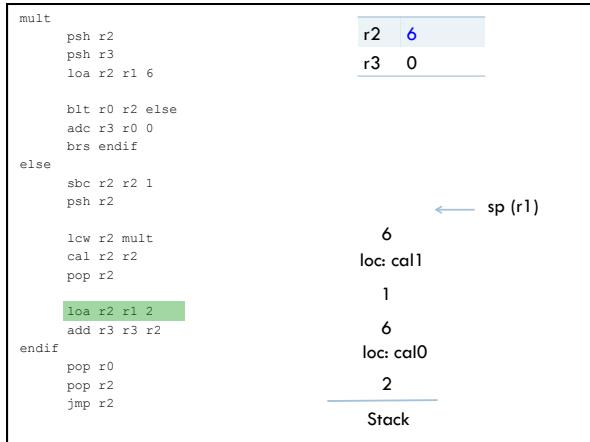
<pre> mult     psh r2     psh r3     loa r2 r1 6      blt r0 r2 else     adc r3 r0 0     brs endif else     sbc r2 r2 1     psh r2      lcw r2 mult     cal r2 r2     pop r2      loa r2 r1 2     add r3 r3 r2 endif     pop r0     pop r2     jmp r2 </pre>	<table border="1" style="margin-bottom: 10px;"> <tr><td>r2</td><td>0</td></tr> <tr><td>r3</td><td>0</td></tr> </table> <p>We don't actually care about this value, but need to get it off the stack to cleanup... just put it in r0, which throws it away</p> <p>loc: call1 ← sp (r1)</p> <table border="1" style="margin-top: 10px;"> <tr><td>0</td></tr> <tr><td>6</td></tr> <tr><td>loc: call1</td></tr> <tr><td>1</td></tr> <tr><td>6</td></tr> <tr><td>loc: call0</td></tr> <tr><td>2</td></tr> </table> <p>Stack</p>	r2	0	r3	0	0	6	loc: call1	1	6	loc: call0	2
r2	0											
r3	0											
0												
6												
loc: call1												
1												
6												
loc: call0												
2												
<pre> mult     psh r2     psh r3     loa r2 r1 6      blt r0 r2 else     adc r3 r0 0     brs endif else     sbc r2 r2 1     psh r2      lcw r2 mult     cal r2 r2     pop r2      loa r2 r1 2     add r3 r3 r2 endif     pop r0     pop r2     jmp r2 </pre>	<table border="1" style="margin-bottom: 10px;"> <tr><td>r2</td><td>0</td></tr> <tr><td>r3</td><td>0</td></tr> </table> <p>loc: call1 ← sp (r1)</p> <table border="1" style="margin-top: 10px;"> <tr><td>0</td></tr> <tr><td>6</td></tr> <tr><td>loc: call1</td></tr> <tr><td>1</td></tr> <tr><td>6</td></tr> <tr><td>loc: call0</td></tr> <tr><td>2</td></tr> </table> <p>Stack</p>	r2	0	r3	0	0	6	loc: call1	1	6	loc: call0	2
r2	0											
r3	0											
0												
6												
loc: call1												
1												
6												
loc: call0												
2												
<pre> mult     psh r2     psh r3     loa r2 r1 6      blt r0 r2 else     adc r3 r0 0     brs endif else     sbc r2 r2 1     psh r2      lcw r2 mult     cal r2 r2     pop r2      loa r2 r1 2     add r3 r3 r2 endif     pop r0     pop r2     jmp r2 </pre>	<table border="1" style="margin-bottom: 10px;"> <tr><td>r2</td><td>loc: call1</td></tr> <tr><td>r3</td><td>0</td></tr> </table> <p>loc: call1 ← sp (r1)</p> <table border="1" style="margin-top: 10px;"> <tr><td>0</td></tr> <tr><td>6</td></tr> <tr><td>loc: call1</td></tr> <tr><td>1</td></tr> <tr><td>6</td></tr> <tr><td>loc: call0</td></tr> <tr><td>2</td></tr> </table> <p>Stack</p>	r2	loc: call1	r3	0	0	6	loc: call1	1	6	loc: call0	2
r2	loc: call1											
r3	0											
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6												
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2												
<pre> mult     psh r2     psh r3     loa r2 r1 6      blt r0 r2 else     adc r3 r0 0     brs endif else     sbc r2 r2 1     psh r2      lcw r2 mult     cal r2 r2     pop r2      loa r2 r1 2     add r3 r3 r2 endif     pop r0     pop r2     jmp r2 </pre>	<table border="1" style="margin-bottom: 10px;"> <tr><td>r2</td><td>loc: call1</td></tr> <tr><td>r3</td><td>0</td></tr> </table> <p>loc: call1 ← sp (r1)</p> <table border="1" style="margin-top: 10px;"> <tr><td>0</td></tr> <tr><td>6</td></tr> <tr><td>loc: call1</td></tr> <tr><td>1</td></tr> <tr><td>6</td></tr> <tr><td>loc: call0</td></tr> <tr><td>2</td></tr> </table> <p>Stack</p>	r2	loc: call1	r3	0	0	6	loc: call1	1	6	loc: call0	2
r2	loc: call1											
r3	0											
0												
6												
loc: call1												
1												
6												
loc: call0												
2												

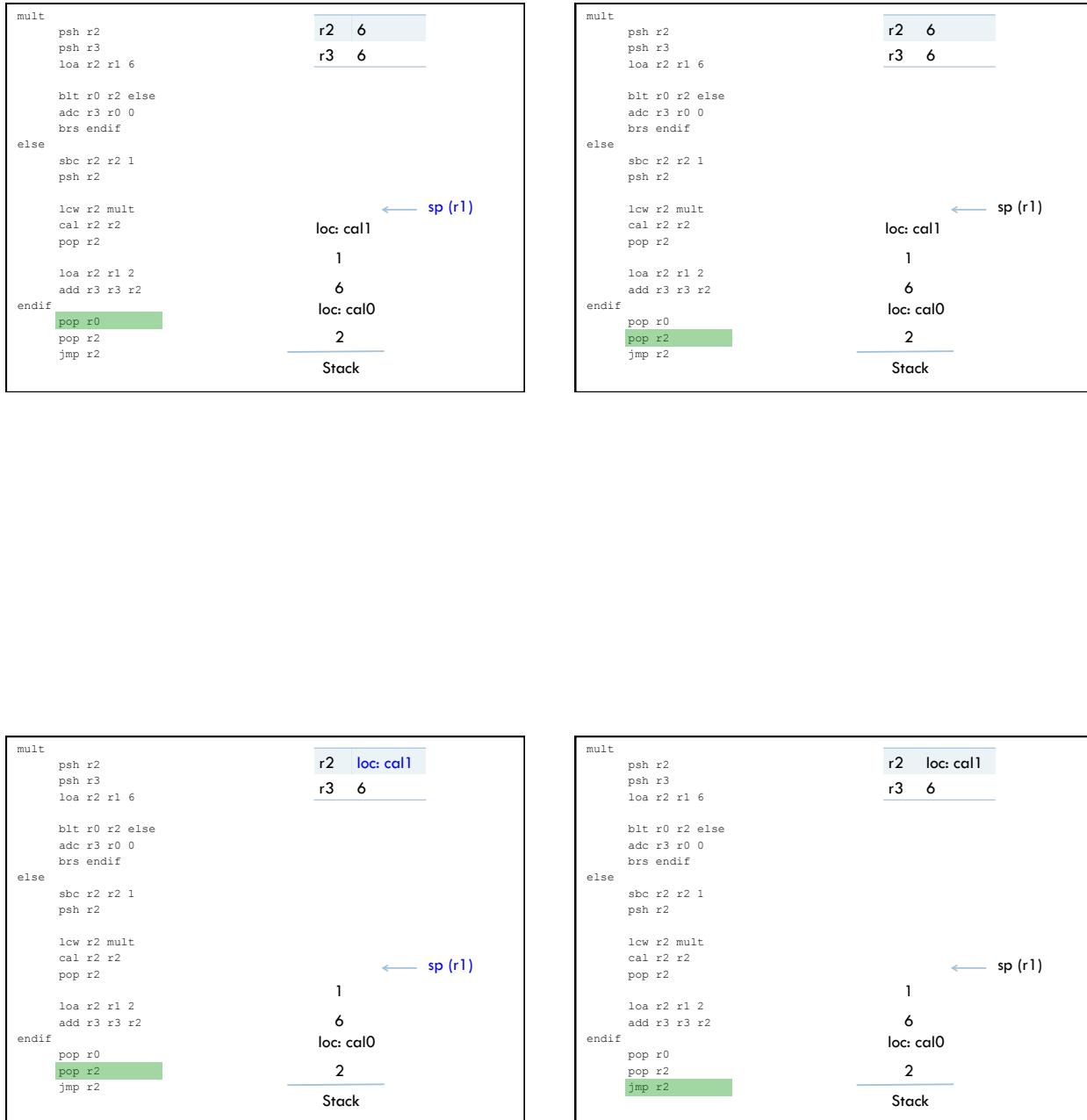
mult	r2	loc: call1
psh r2		
psh r3		
loa r2 r1 6		
blt r0 r2 else		
adc r3 r0 0		
brs endif		
else		
sbc r2 r2 1		
psh r2		
lcw r2 mult		
cal r2 r2		
pop r2		
loa r2 r1 2		
add r3 r3 r2		
endif		
pop r0		
pop r2		
jmp r2		
Returning with answer in r3		
	sp (r1)	
	0	
	6	
	loc: call1	
	1	
	6	
	loc: cal0	
	2	
	Stack	

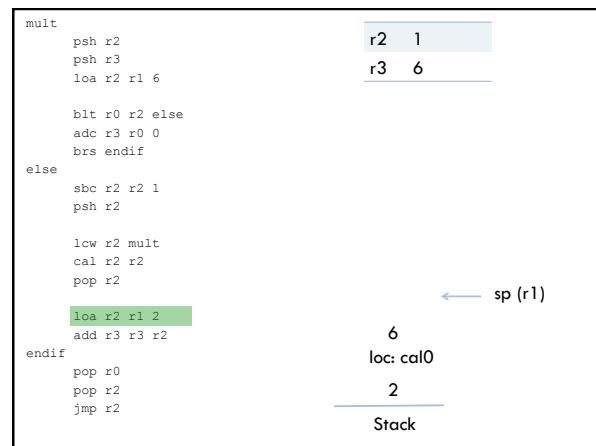
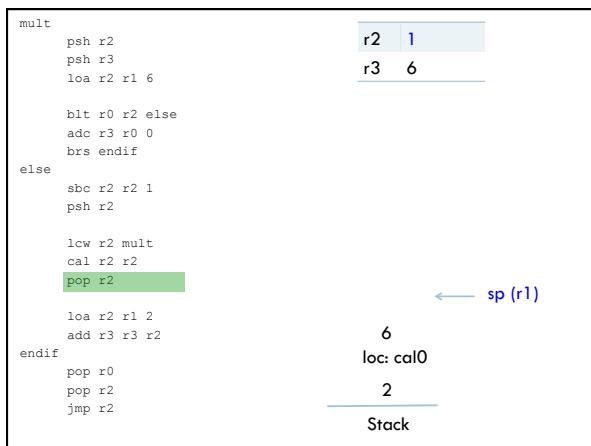
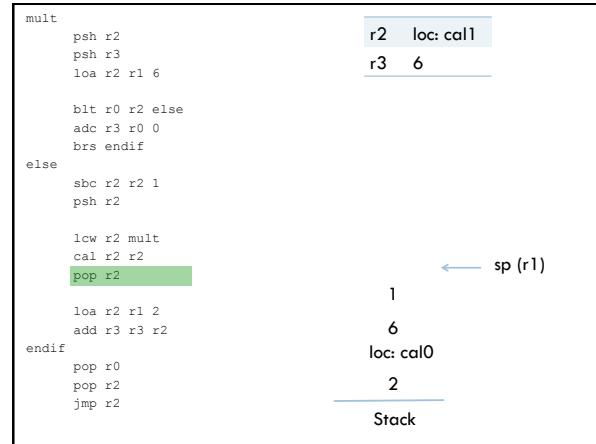
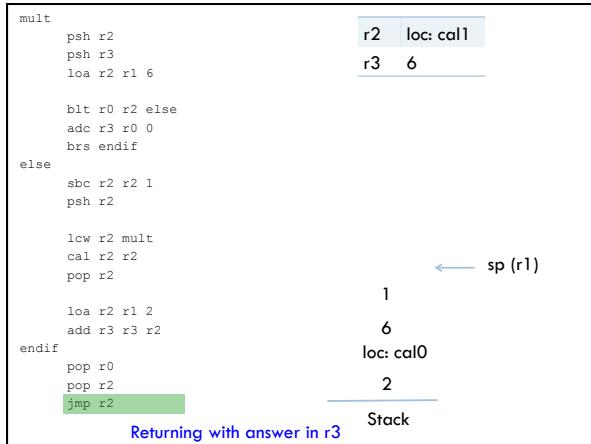
mult	r2	loc: call1
psh r2		
psh r3		
loa r2 r1 6		
blt r0 r2 else		
adc r3 r0 0		
brs endif		
else		
sbc r2 r2 1		
psh r2		
lcw r2 mult		
cal r2 r2		
pop r2		
loa r2 r1 2		
add r3 r3 r2		
endif		
pop r0		
pop r2		
jmp r2		
Returning with answer in r3		
	sp (r1)	
	0	
	6	
	loc: call1	
	1	
	6	
	loc: cal0	
	2	
	Stack	

mult	r2	0
psh r2		
psh r3		
loa r2 r1 6		
blt r0 r2 else		
adc r3 r0 0		
brs endif		
else		
sbc r2 r2 1		
psh r2		
lcw r2 mult		
cal r2 r2		
pop r2		
loa r2 r1 2		
add r3 r3 r2		
endif		
pop r0		
pop r2		
jmp r2		
Returning with answer in r3		
	sp (r1)	
	6	
	loc: call1	
	1	
	6	
	loc: cal0	
	2	
	Stack	

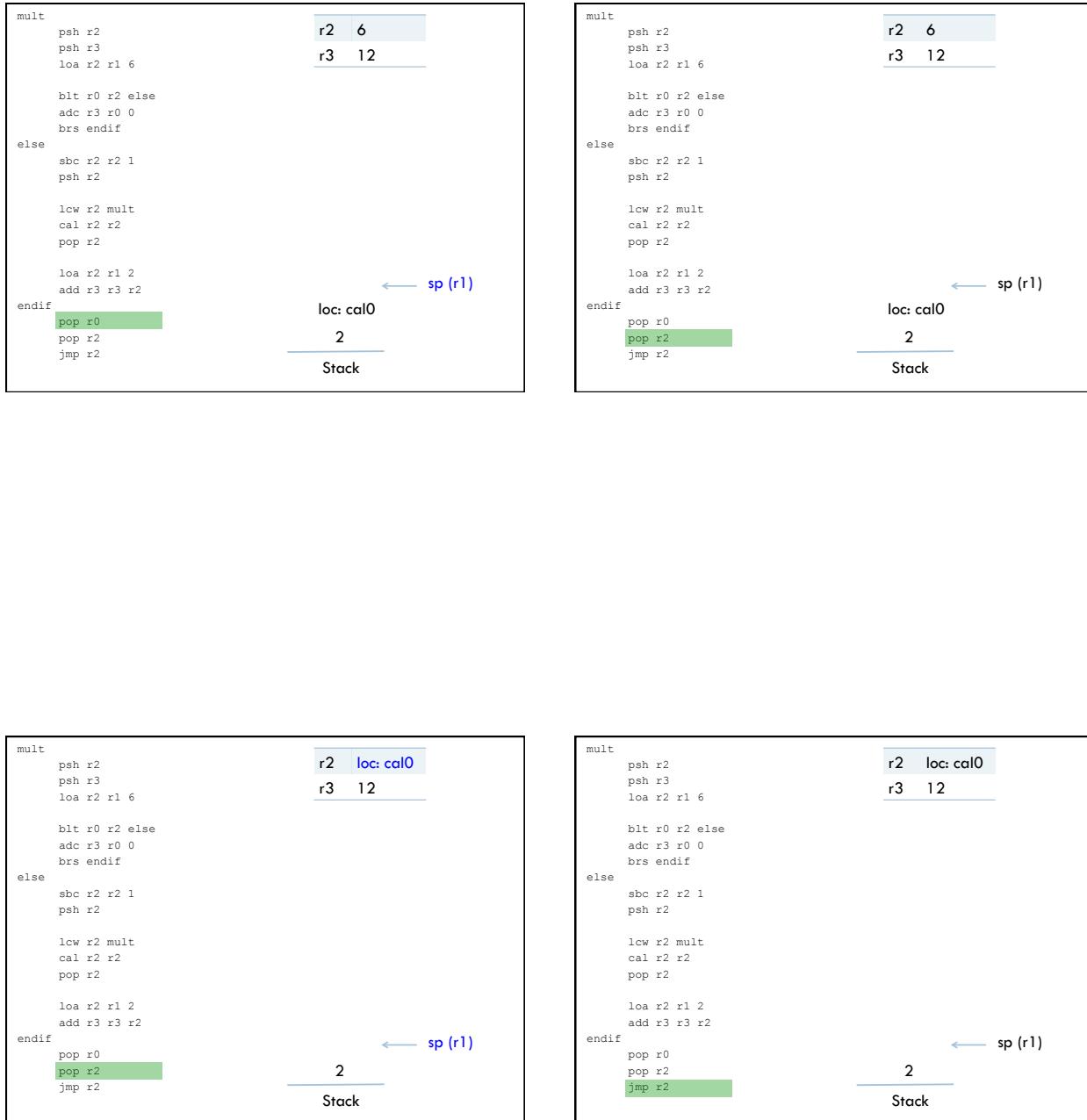
mult	r2	0
psh r2		
psh r3		
loa r2 r1 6		
blt r0 r2 else		
adc r3 r0 0		
brs endif		
else		
sbc r2 r2 1		
psh r2		
lcw r2 mult		
cal r2 r2		
pop r2		
loa r2 r1 2		
add r3 r3 r2		
endif		
pop r0		
pop r2		
jmp r2		
Returning with answer in r3		
	sp (r1)	
	6	
	loc: call1	
	1	
	6	
	loc: cal0	
	2	
	Stack	

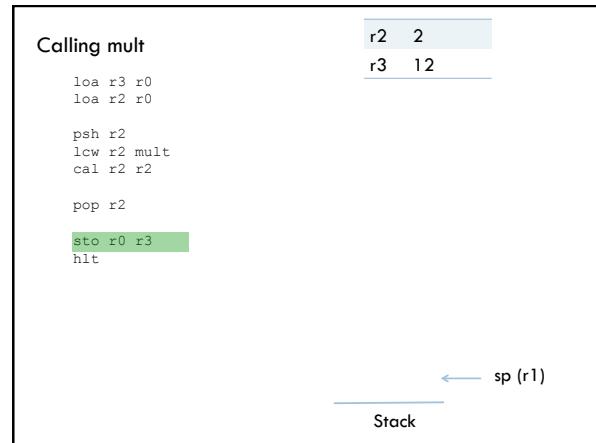
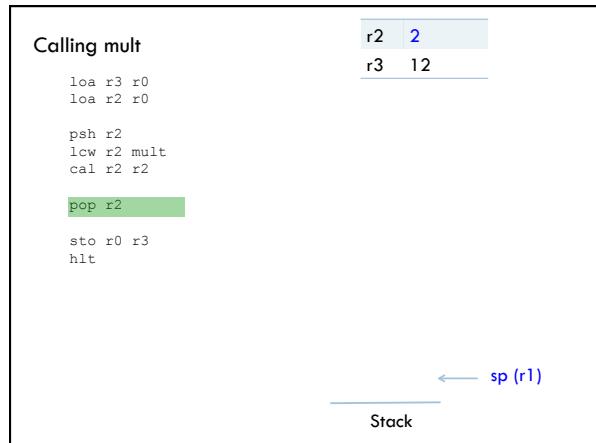
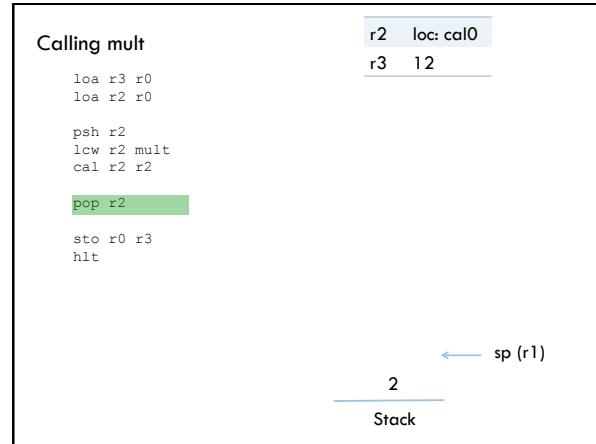
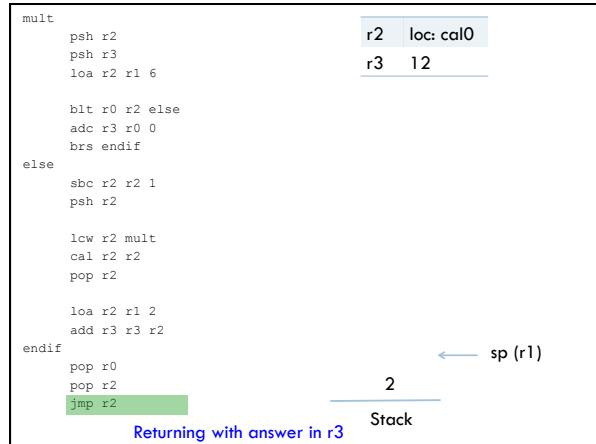












**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r2
sto r0 r3
hlt

```

Print the answer: 12!

r2	2
r3	12

← sp (r1)  
Stack

**Calling mult**

```

loa r3 r0
loa r2 r0

psh r2
lcw r2 mult
cal r2 r2

pop r2
sto r0 r3
hlt

```

Print the answer: 12!

r2	2
r3	12

← sp (r1)  
Stack

**Run mult.a41 in simulator****CS41B programming advice**

1. Match your push and pops
2. Follow the register conventions
3. Develop code incrementally
4. Debugging: write out stack, registers, etc. on paper and compare against system execution

## Examples from this lecture

<http://www.cs.pomona.edu/~dkauchak/classes/cs52/examples/cs41b/>