

# BOOSTING

David Kauchak  
CS451 – Fall 2013

## Admin

Final project

## Ensemble learning

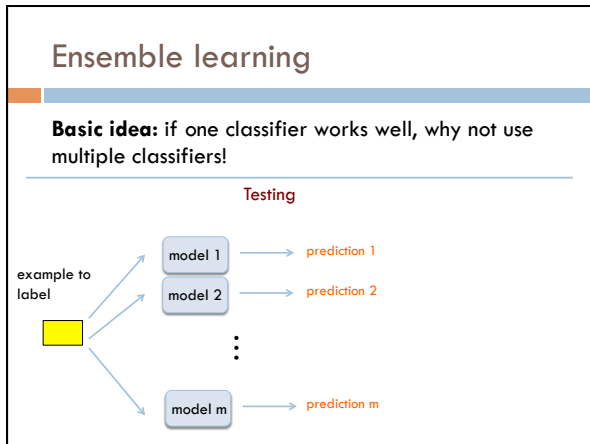
**Basic idea:** if one classifier works well, why not use multiple classifiers!

## Ensemble learning

**Basic idea:** if one classifier works well, why not use multiple classifiers!

Training


```
graph LR; TD[Training Data] --> LA1[learning alg]; TD --> LA2[learning alg]; TD --> LA3[learning alg]; TD --> LA4[learning alg]; LA1 --> M1[model 1]; LA2 --> M2[model 2]; LA3 --> Dots[...]; LA4 --> Mm[model m];
```



### Idea 4: boosting

training data			"training" data 2			"training" data 3		
Data	Label	Weight	Data	Label	Weight	Data	Label	Weight
■	0	0.2	■	0	0.1	■	0	0.05
■	0	0.2	■	0	0.1	■	0	0.2
■	1	0.2	■	1	0.4	■	1	0.2
■	1	0.2	■	1	0.1	■	1	0.05
■	0	0.2	■	0	0.3	■	0	0.5

### "Strong" learner




Given

- a reasonable amount of training data
- a target error rate  $\epsilon$
- a failure probability  $p$

A **strong learning algorithm** will produce a classifier with error rate  $< \epsilon$  with probability  $1-p$

### "Weak" learner








Given


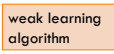


- a reasonable amount of training data
- a failure probability  $p$

A **weak learning algorithm** will produce a classifier with error rate  $< 0.5$  with probability  $1-p$

Weak learners are much easier to create!

### weak learners for boosting

Data	Label	Weight
	0	0.2
	0	0.2
	1	0.2
	1	0.2
	0	0.2

Which of our algorithms can handle weights?

Need a weak learning algorithm that can handle **weighted** examples

### boosting: basic algorithm

**Training:**  
start with equal example weights

for some number of iterations:

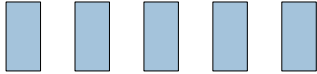
- learn a weak classifier and save
- change the example weights

**Classify:**


- get prediction from all learned weak classifiers
- weighted vote based on how well the weak classifier did when it was trained

### boosting basics

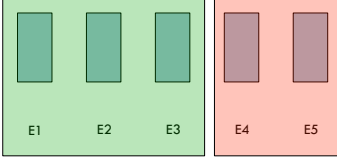
Start with equal weighted examples

Weights: 

Examples: E1 E2 E3 E4 E5

Learn a weak classifier: 

### Boosting

Weights: 

Examples: E1 E2 E3 E4 E5

classified correct (green) / classified incorrect (red)

We want to reweight the examples and then learn another weak classifier

How should we change the example weights?

### Boosting

Weights:

Examples: E1 E2 E3 E4 E5

- decrease the weight for those we're getting correct
- increase the weight for those we're getting incorrect

### Boosting

Weights:

Examples: E1 E2 E3 E4 E5

Learn another weak classifier:

### Boosting

Weights:

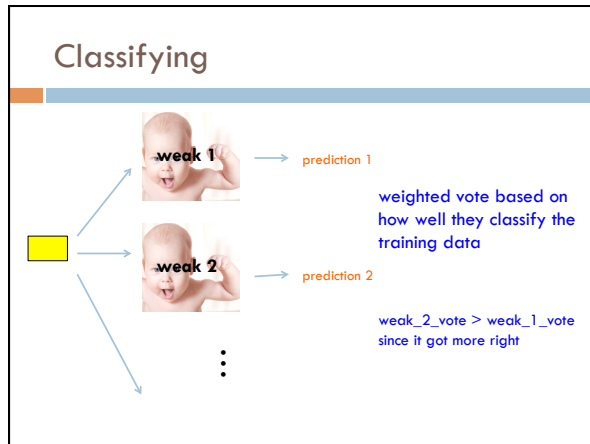
Examples: E1 E2 E3 E4 E5

### Boosting

Weights:

Examples: E1 E2 E3 E4 E5

- decrease the weight for those we're getting correct
- increase the weight for those we're getting incorrect



## Notation

$x_i$	example $i$ in the training data
$w_i$	weight for example $i$ , we will enforce: $w_i \geq 0$ $\sum_{i=1}^n w_i = 1$
$\text{classifier}_k(x_i)$	+1/-1 prediction of classifier $k$ example $i$

## AdaBoost: train

for  $k = 1$  to iterations:

- $\text{classifier}_k =$  learn a weak classifier based on weights
- calculate weighted error for this classifier

$$\epsilon_k = \sum_{i=1}^n w_i * \mathbb{1}[\text{label}_i \neq \text{classifier}_k(x_i)]$$

- calculate "score" for this classifier:

$$\alpha_k = \frac{1}{2} \log\left(\frac{1 - \epsilon_k}{\epsilon_k}\right)$$

- change the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * \text{label}_i * \text{classifier}_k(x_i))$$

## AdaBoost: train

$\text{classifier}_k =$  learn a weak classifier based on weights

weighted error for this classifier is:

$$\epsilon_k = \sum_{i=1}^n w_i * \mathbb{1}[\text{label}_i \neq \text{classifier}_k(x_i)]$$

What does this say?

### AdaBoost: train

classifier<sub>k</sub> = learn a weak classifier based on weights

weighted error for this classifier is:

$$\epsilon_k = \sum_{i=1}^n w_i * \mathbb{1}[\text{label}_i \neq \text{prediction}_k(x_i)]$$

What is the range of possible values?

prediction

did we get the example wrong

weighted sum of the errors/mistakes

### AdaBoost: train

classifier<sub>k</sub> = learn a weak classifier based on weights

weighted error for this classifier is:

$$\epsilon_k = \sum_{i=1}^n w_i * \mathbb{1}[\text{label}_i \neq \text{prediction}_k(x_i)]$$

Between 0, if we get all examples right, and 1, if we get them all wrong

prediction

did we get the example wrong

weighted sum of the errors/mistakes

### AdaBoost: train

classifier<sub>k</sub> = learn a weak classifier based on weights

“score” or weight for this classifier is:

$$\alpha_k = \frac{1}{2} \log \left( \frac{1 - \epsilon_i}{\epsilon_i} \right)$$

What does this look like (specifically for errors between 0 and 1)?

### AdaBoost: train

$$\alpha_k = \frac{1}{2} \log \left( \frac{1 - \epsilon_i}{\epsilon_i} \right)$$

- ranges from 1 to -1
- errors of 50% = 0

## AdaBoost: classify

$$\text{classify}(x) = \text{sign} \left( \sum_{k=1}^{\text{iterations}} \alpha_k * \text{classifier}_k(x) \right)$$

What does this do?

## AdaBoost: classify

$$\text{classify}(x) = \text{sign} \left( \sum_{k=1}^{\text{iterations}} \alpha_k * \text{classifier}_k(x) \right)$$

The weighted vote of the learned classifiers weighted by  $\alpha$  (remember  $\alpha$  varies from 1 to -1 training error)

What happens if a classifier has error >50%

## AdaBoost: classify

$$\text{classify}(x) = \text{sign} \left( \sum_{k=1}^{\text{iterations}} \alpha_k * \text{classifier}_k(x) \right)$$

The weighted vote of the learned classifiers weighted by  $\alpha$  (remember  $\alpha$  varies from 1 to -1 training error)

We actually vote the opposite!

## AdaBoost: train, updating the weights

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * \text{label}_i * \text{classifier}_k(x_i))$$

Remember, we want to enforce:

$$w_i \geq 0$$

$$\sum_{i=1}^n w_i = 1$$

$Z$  is called the **normalizing constant**. It is used to make sure that the weights sum to 1

What should it be?

### AdaBoost: train

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

Remember, we want to enforce:

$$w_i \geq 0$$

$$\sum_{i=1}^n w_i = 1$$

normalizing constant (i.e. the sum of the "new"  $w_i$ ):

$$Z = \sum_{i=1}^n w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

### AdaBoost: train

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

What does this do?

### AdaBoost: train

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

correct
positive
  

incorrect
negative
  

correct
?
  

incorrect

### AdaBoost: train

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

correct
positive
  

incorrect
negative
  

correct
small value
  

incorrect
large value

Note: only change weights based on current classifier (not all previous classifiers)



### AdaBoost: train

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

What does the  $\alpha$  do?

### AdaBoost: train

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

What does the  $\alpha$  do?

If the classifier was good (<50% error)  $\alpha$  is positive:  
trust classifier output and move as normal

If the classifier was back (>50% error)  $\alpha$  is negative  
classifier is so bad, consider opposite prediction of classifier

### AdaBoost: train

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

correct positive  
incorrect negative

correct small value  
incorrect large value

If the classifier was good (<50% error)  $\alpha$  is positive  
If the classifier was back (>50% error)  $\alpha$  is negative

### AdaBoost justification

update the example weights

$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

Does this look like anything we've seen before?

### AdaBoost justification

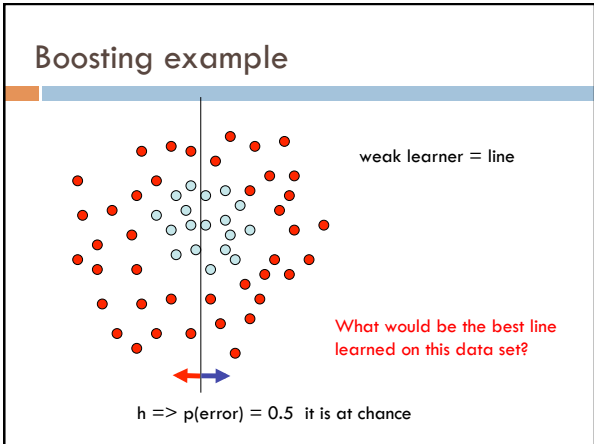
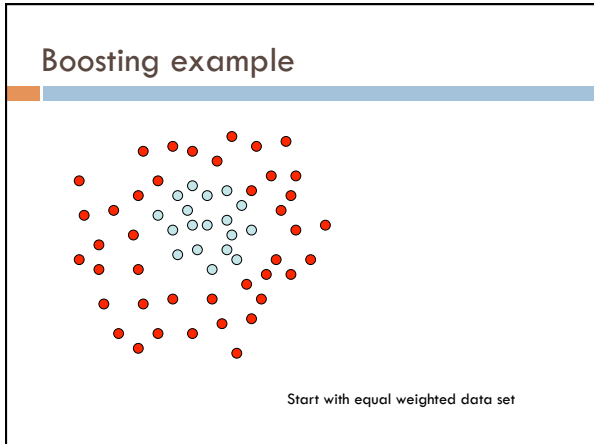
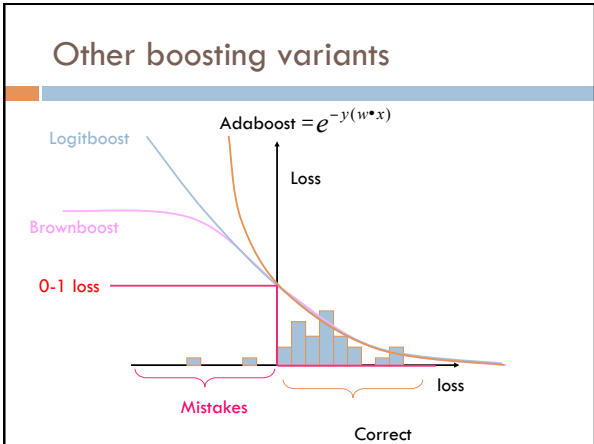
update the example weights

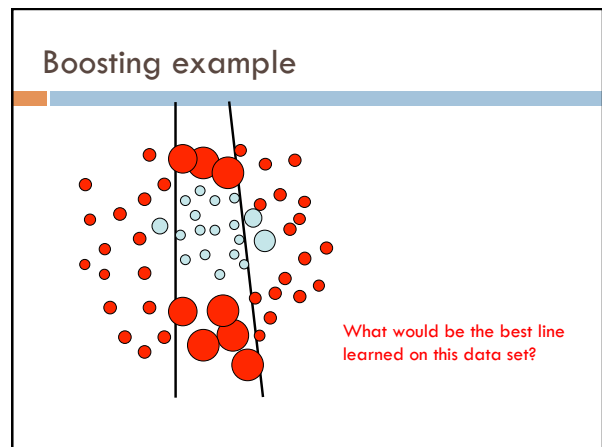
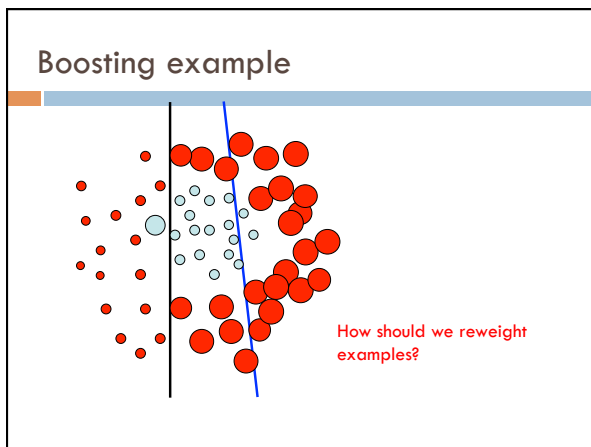
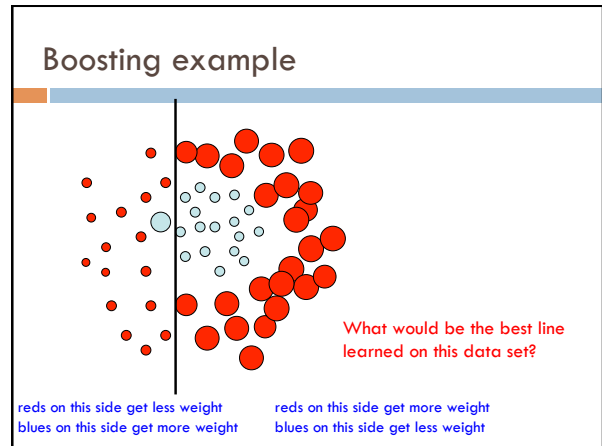
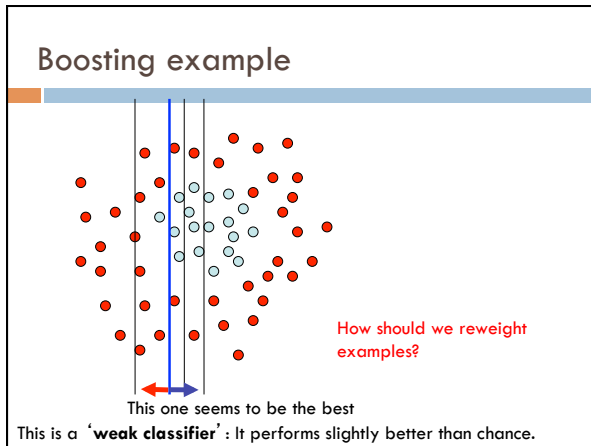
$$w_i = \frac{1}{Z} w_i \exp(-\alpha_k * label_i * classifier_k(x_i))$$

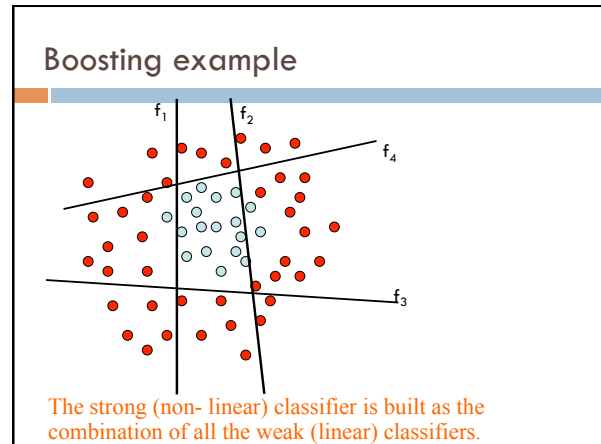
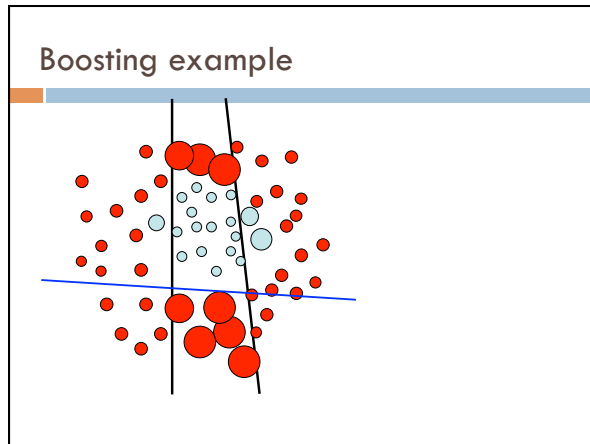
Exponential loss!

$$l(y, y') = \exp(-y y')$$

AdaBoost turns out to be another approach for minimizing the exponential loss!







### AdaBoost: train

for  $k = 1$  to iterations:

- classifier<sub>k</sub> = learn a weak classifier based on weights
- weighted error for this classifier is:
- "score" or weight for this classifier is:
- change the example weights

What can we use as a classifier?

### AdaBoost: train

for  $k = 1$  to iterations:

- classifier<sub>k</sub> = learn a weak classifier based on weights
- weighted error for this classifier is:
- "score" or weight for this classifier is:
- change the example weights

- Anything that can train on weighted examples
- For most applications, must be fast!

Why?

## AdaBoost: train

for  $k = 1$  to *iterations*:

- $\text{classifier}_k = \text{learn a weak classifier based on weights}$
- weighted error for this classifier is:
- "score" or weight for this classifier is:
- change the example weights
- Anything that can train on weighted examples
- For most applications, must be fast!
  - Each iteration we have to train a new classifier

## Boosted decision stumps

One of the most common classifiers to use is a decision tree:

- can use a shallow (2-3 level tree)
- even more common is a 1-level tree
  - called a **decision stump** ☺
  - asks a question about a single feature

What does the decision boundary look like for a decision stump?

## Boosted decision stumps

One of the most common classifiers to use is a decision tree:

- can use a shallow (2-3 level tree)
- even more common is a 1-level tree
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  - asks a question about a single feature

What does the decision boundary look like for boosted decision stumps?

## Boosted decision stumps

One of the most common classifiers to use is a decision tree:

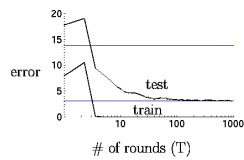
- can use a shallow (2-3 level tree)
- even more common is a 1-level tree
  - called a **decision stump** ☺
  - asks a question about a single feature

- **Linear classifier!**
- Each stump defines the weight for that dimension
  - If you learn multiple stumps for that dimension then it's the weighted average

## Boosting in practice

Very successful on a wide range of problems

One of the keys is that boosting tends not to overfit, even for a large number of iterations

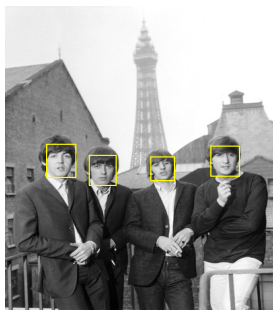


Using <10,000 training examples can fit >2,000,000 parameters!

## Adaboost application example: face detection



## Adaboost application example: face detection



### Rapid Object Detection using a Boosted Cascade of Simple Features


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[Rapid object detection using a boosted cascade of simple features](#)  
P. Viola, M. Jones - ... Vision and Pattern Recognition, 2001. CVPR ..., 2001 - [ieeexplore.ieee.org](#)  
... overlap. Each partition yields a single final **detection**. The ... set. Experiments on a  
Real-World Test Set We tested our system on the MIT+CMU frontal **face** test set [11].  
This set consists of 130 images with 507 labeled frontal **faces**. A ...  
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[Rapid object detection using a boosted cascade of simple features](#)  
 P. Viola, M. Jones - ... Vision and Pattern Recognition, 2001. CVPR. ... 2001 - leexplora.leece.org  
 ... overlap. Each partition yields a single final **detection**. The ... set. Experiments on a Real-World Test Set We tested our system on the MIT+CMU frontal **face** test set [ 11]. This set consists of 130 images with 507 labeled frontal **faces**. A ...  
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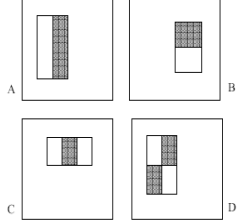
To give you some context of importance:

[The anatomy of a large-scale hypertextual Web search engine](#)   
 S. Brin, L. Page - Computer networks and ISDN systems, 1998 - Elsevier  
 ... This is largely because they all have high **PageRank**. ... However, once the system was running smoothly, S. Brin, L. Page Computer Networks and ISDN Systems 30 ... Google employs a number of techniques to improve search quality including **page rank**, anchor text, and proximity ...  
 Cited by 11070 Related articles All 349 versions Cite Save

or:

[Modeling word burstiness using the Dirichlet distribution](#)  
 RE Madsen, D. Kauchak, C. Elkan - Proceedings of the 22nd international ..., 2005 - dl.acm.org  
 Abstract Multinomial distributions are often used to model text documents. However, they do not capture well the phenomenon that words in a document tend to appear in bursts: if a word appears once, it is more likely to appear again. In this paper, we propose the ...  
 Cited by 148 Related articles All 34 versions Cite Save

## “weak” learners

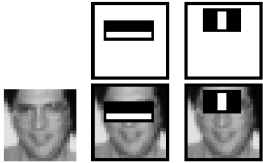



4 Types of “Rectangle filters” (Similar to Haar wavelet Papageorgiou, et al.)

Based on 24x24 grid:  
 160,000 features to choose from

$$g(x) = \text{sum(WhiteArea)} - \text{sum(BlackArea)}$$

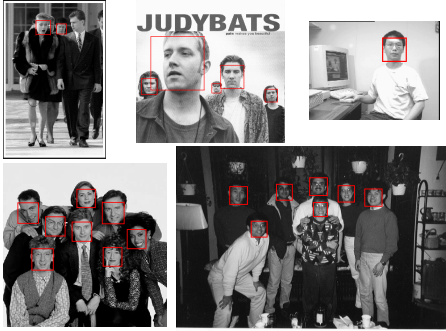
## “weak” learners



$$F(x) = \alpha_1 f_1(x) + \alpha_2 f_2(x) + \dots$$

$$f_i(x) = \begin{cases} 1 & \text{if } g_i(x) > \theta_i \\ -1 & \text{otherwise} \end{cases}$$

## Example output



### Solving other "Face" Tasks

Facial Feature Localization      Profile Detection

Demographic Analysis

### "weak" classifiers learned

### Bagging vs Boosting

Journal of Artificial Intelligence Research 11 (1999) 169-198      Submitted 1/99; published 8/99

#### Popular Ensemble Methods: An Empirical Study

**David Opitz**  
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<http://arxiv.org/pdf/1106.0257.pdf>

