Language acquisition

- http://www.youtube.com/watch?v=RE4ce4mexrU

LANGUAGE MODELING: SMOOTHING

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CS457 – Fall 2011

some slides adapted from Jason Eisner

Admin

- Assignment 2 out
  - bigram language modeling
  - Java
  - Can work with partners
    - Anyone looking for a partner?
  - Due Wednesday 10/5
  - Style/commenting (JavaDoc)
  - Some advice
    - Start now!
    - Spend 1-2 hours working out an example by hand (you can check your answers with me)
    - HashMap

Admin

- Our first quiz next Tuesday (10/4)
  - In-class (~30 min.)
  - Topics
    - corpus analysis
    - regular expressions
    - probability
    - language modeling
  - Open book
    - we'll try it out for this one
    - better to assume closed book (30 minutes goes by fast!)
  - 7.5% of your grade

Today

Today

- Take home ideas:
  - Key idea of smoothing is to redistribute the probability to handle less seen (or never seen) events
    - Still must always maintain a true probability distribution
  - Lots of ways of smoothing data
  - Should take into account features in your data!

Smoothing

What if our test set contains the following sentence, but one of the trigrams never occurred in our training data?

P(I think today is a good day to be me) =

P(I | <start> <start>) x
P(think | <start> I) x
P(today | I think) x
P(is | think today) x
P(a | today is) x
P(good | is a) x
…

If any of these has never been seen before, prob = 0!

Smoothing

P(I think today is a good day to be me) =

P(I | <start> <start>) x
P(think | <start> I) x
P(today | I think) x
P(is | think today) x
P(a | today is) x
P(good | is a) x
…

These probability estimates may be inaccurate. Smoothing can help reduce some of the noise.

Add-lambda smoothing

- A large dictionary makes novel events too probable.
- add λ = 0.01 to all counts

see the abacus	1	1/3	1.01	1.01/203
see the abbot	0	0/3	0.01	0.01/203
see the abduct	0	0/3	0.01	0.01/203
see the above	2	2/3	2.01	2.01/203
see the Abram	0	0/3	0.01	0.01/203
…			0.01	0.01/203
see the zygote	0	0/3	0.01	0.01/203
Total	3	3/3	203	

Vocabulary

- n-gram language modeling assumes we have a fixed vocabulary
  - why?
- Whether implicit or explicit, an n-gram language model is defined over a finite, fixed vocabulary
- What happens when we encounter a word not in our vocabulary (Out Of Vocabulary)?
  - If we don't do anything, prob = 0
  - Smoothing doesn't really help us with this!

Vocabulary

- To make this explicit, smoothing helps us with…

all entries in our vocabulary

see the abacus	1	1.01
see the abbot	0	0.01
see the abduct	0	0.01
see the above	2	2.01
see the Abram	0	0.01
…		0.01
see the zygote	0	0.01

Vocabulary

- and…

Vocabulary	Counts	Smoothed counts
a	10	10.01
able	1	1.01
about	2	2.01
account	0	0.01
acid	0	0.01
across	3	3.01
…	…	…
young	1	1.01
zebra	0	0.01

How can we have words in our vocabulary we've never seen before?

Vocabulary

- Choosing a vocabulary: ideas?
  - Grab a list of English words from somewhere
  - Use all of the words in your training data
  - Use some of the words in your training data
    - for example, all those the occur more than k times
- Benefits/drawbacks?
  - Ideally your vocabulary should represents words you're likely to see
  - Too many words: end up washing out your probability estimates (and getting poor estimates)
  - Too few: lots of out of vocabulary

Vocabulary

- No matter your chosen vocabulary, you're still going to have out of vocabulary (OOV)
- How can we deal with this?
  - Ignore words we've never seen before
    - Somewhat unsatisfying, though can work depending on the application
    - Probability is then dependent on how many in vocabulary words are seen in a sentence/text
  - Use a special symbol for OOV words and estimate the probability of out of vocabulary

Out of vocabulary

- Add an extra word in your vocabulary to denote OOV (<OOV>, <UNK>)
- Replace all words in your training corpus not in the vocabulary with <UNK>
  - You'll get bigrams, trigrams, etc with <UNK>
    - p(<UNK> | "I am")
    - p(fast | "I <UNK>")
- During testing, similarly replace all OOV with <UNK>

Choosing a vocabulary

- A common approach (and the one we'll use for the assignment):
  - Replace the first occurrence of each word by <UNK> in a data set
  - Estimate probabilities normally
- Vocabulary then is all words that occurred two or more times
- This also discounts all word counts by 1 and gives that probability mass to <UNK>

Storing the table

How are we storing this table?
Should we store all entries?

see the abacus	1	1/3	1.01	1.01/203
see the abbot	0	0/3	0.01	0.01/203
see the abduct	0	0/3	0.01	0.01/203
see the above	2	2/3	2.01	2.01/203
see the Abram	0	0/3	0.01	0.01/203
…			0.01	0.01/203
see the zygote	0	0/3	0.01	0.01/203
Total	3	3/3	203	

Storing the table

- Hashtable
  - fast retrieval
  - fairly good memory usage
- Only store those entries of things we've seen
  - for example, we don't store $|V|^3$ trigrams
- For trigrams we can:
  - Store one hashtable with bigrams as keys
  - Store a hashtable of hashtables (I'm recommending this)

Storing the table: add-lambda smoothing

- For those we've seen before:

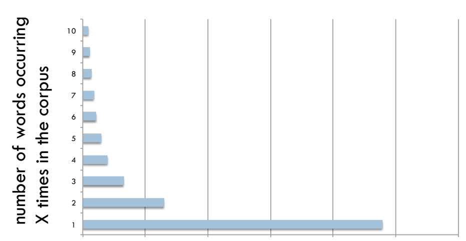
$$P(c \mid ab) = \frac{C(abc) + \lambda}{C(ab) + \lambda V}$$

- Unseen n-grams: p(z|ab) = ?

$$P(z \mid ab) = \frac{\lambda}{C(ab) + \lambda V}$$

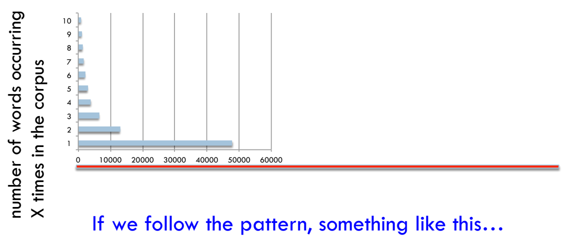
Store the lower order counts
(or probabilities)

How common are novel events?



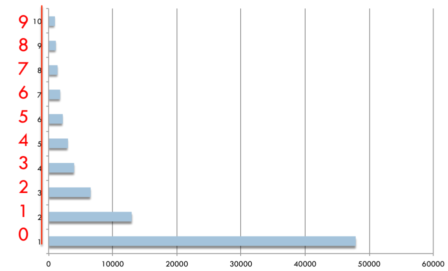
How likely are novel/unseen events?

How common are novel events?



If we follow the pattern, something like this…

Good-Turing estimation



Good-Turing estimation

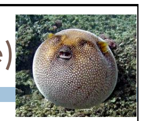
- N_c = number of words/bigrams occurring c times
- Estimate the probability of novel events as:

$$p(unseen) = \frac{N_1}{Total\_words}$$

- Replace MLE counts for things with count c:

$$c^* = (c+1)\underbrace{\frac{N_{c+1}}{N_c}}_{\text{scale down the next frequency up}}$$

Good-Turing (classic example)



- Imagine you are fishing
  - 8 species: carp, perch, whitefish, trout, salmon, eel, catfish, bass
- You have caught
  - 10 carp, 3 perch, 2 whitefish, 1 trout, 1 salmon, 1 eel = 18 fish
- How likely is it that the next fish caught is from a new species (one not seen in our previous catch)?

$$p(unseen) = \frac{N_1}{Total\_words} = \frac{3}{18}$$

Good-Turing (classic example)

- Imagine you are fishing
  - 8 species: carp, perch, whitefish, trout, salmon, eel, catfish, bass
- You have caught
  - 10 carp, 3 perch, 2 whitefish, 1 trout, 1 salmon, 1 eel = 18 fish
- How likely is it that next species is trout?

$$c^* = (c+1)\frac{N_{c+1}}{N_c} = 2 * \frac{1}{3} = 0.67$$

$$\frac{0.67}{18}$$

Good-Turing (classic example)

- Imagine you are fishing
  - 8 species: carp, perch, whitefish, trout, salmon, eel, catfish, bass
- You have caught
  - 10 carp, 3 perch, 2 whitefish, 1 trout, 1 salmon, 1 eel = 18 fish
- How likely is it that next species is perch?

$$c^* = (c+1)\frac{N_{c+1}}{N_c}$$

N_4 is 0!

Nice idea, but kind of a pain to implement in practice

Problems with frequency based smoothing

- The following bigrams have never been seen:

p(X | San)　　　　p(X | ate)

Which would add-lambda pick as most likely?

Which would you pick?

Witten-Bell Discounting

- Some words are more likely to be followed by new words

San	Diego Francisco Luis Jose Marcos	ate	food apples bananas hamburgers a lot for two grapes …

Witten-Bell Discounting

- Probability mass is shifted around, depending on the context of words
- If $P(w_i \mid w_{i-1},\ldots,w_{i-m}) = 0$, then the smoothed probability $P_{WB}(w_i \mid w_{i-1},\ldots,w_{i-m})$ is higher if the sequence $w_{i-1},\ldots,w_{i-m}$ occurs with many different words w_k

Witten-Bell Smoothing

- For bigrams
  - $T(w_{i-1})$ is the number of different words (types) that occur to the right of w_{i-1}
  - $N(w_{i-1})$ is the number of times w_{i-1} occurred
  - $Z(w_{i-1})$ is the number of bigrams in the current data set starting with w_{i-1} that do not occur in the training data

Witten-Bell Smoothing

- if $c(w_{i-1},w_i) > 0$

$$P^{WB}(w_i \mid w_{i-1}) = \frac{c(w_{i-1}w_i)}{N(w_{i-1}) + T(w_{i-1})}$$

$$\frac{\text{\# times we saw the bigram}}{\text{\# times } w_{i-1} \text{ occurred} + \text{\# of types to the right of } w_{i-1}}$$

Witten-Bell Smoothing

- If $c(w_{i-1},w_i) = 0$

$$P^{WB}(w_i \mid w_{i-1}) = \frac{T(w_{i-1})}{Z(w_{i-1})(N + T(w_{i-1}))}$$

Problems with frequency based smoothing

- The following trigrams have never been seen:

p(car | see the)   p(zygote | see the)

p(cumquat | see the)

Which would add-lambda pick as most likely?
Good-Turing? Witten-Bell?

Which would you pick?

Better smoothing approaches

- Utilize information in lower-order models
- Interpolation
  - p*(z| x,y) = λ p(z | x, y) + μ p(z | y) + (1- λ - μ)p(z)
  - Combine the probabilities in some linear combination
- Backoff

$$P(z \mid xy) = \begin{cases} \frac{C^*(xyz)}{C(xy)} & \text{if } C(xyz) > k \\ \alpha(xy)P(z \mid y) & \text{otherwise} \end{cases}$$

  - Often k = 0 (or 1)
  - Combine the probabilities by "backing off" to lower models only when we don't have enough information

Smoothing: Simple Interpolation

$$P(z \mid xy) \approx \lambda \frac{C(xyz)}{C(xy)} + \mu \frac{C(yz)}{C(y)} + (1 - \lambda - \mu) \frac{C(z)}{C(\bullet)}$$

- Trigram is very context specific, very noisy
- Unigram is context-independent, smooth
- Interpolate Trigram, Bigram, Unigram for best combination
- How should we determine λ and μ ?

Smoothing: Finding parameter values

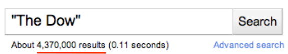
- Just like we talked about before, split training data into training and development
  - can use cross-validation, leave-one-out, etc.
- Try lots of different values for λ, μ on heldout data, pick best
- Two approaches for finding these efficiently
  - EM (expectation maximization)
  - "Powell search" – see Numerical Recipes in C

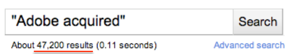
Smoothing: Jelinek-Mercer

- Simple interpolation:

$$P_{smooth}(z \mid xy) = \lambda \frac{C(xyz)}{C(xy)} + (1-\lambda) P_{smooth}(z \mid y)$$

- Should all bigrams be smoothed equally? Which of these is more likely to start an unseen trigram?





Smoothing: Jelinek-Mercer

- Simple interpolation:

$$P_{smooth}(z \mid xy) = \lambda \frac{C(xyz)}{C(xy)} + (1-\lambda) P_{smooth}(z \mid y)$$

- Multiple parameters based on frequency bins: smooth a little after "The Dow", more after "Adobe acquired"

$$P_{smooth}(z \mid xy) = \lambda(C(xy)) \frac{C(xyz)}{C(xy)} + (1-\lambda(C(xy)) P_{smooth}(z \mid y)$$

Smoothing: Jelinek-Mercer continued

$$P_{smooth}(z \mid xy) = \lambda(C(xy)) \frac{C(xyz)}{C(xy)} + (1-\lambda(C(xy)) P_{smooth}(z \mid y)$$

- Bin counts by frequency and assign λs for each bin
- Find λs by cross-validation on held-out data

Backoff models: absolute discounting

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz)-D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

- Subtract some absolute number from each of the counts (e.g. 0.75)
  - How will this affect rare words?
  - How will this affect common words?

Backoff models: absolute discounting

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

- Subtract some absolute number from each of the counts (e.g. 0.75)
  - will have a large effect on low counts (rare words)
  - will have a small effect on large counts (common words)

Backoff models: absolute discounting

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

What is α(xy)?

Backoff models: absolute discounting

see the dog	1
see the cat	2
see the banana	4
see the man	1
see the woman	1
see the car	1

the Dow Jones	10
the Dow rose	5
the Dow fell	5

p(rose | the Dow) = ?

p(jumped | the Dow) = ?

p(cat | see the) = ?

p(puppy | see the) = ?

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

Backoff models: absolute discounting

see the dog	1
see the cat	2
see the banana	4
see the man	1
see the woman	1
see the car	1

p(cat | see the) = ?

$$\frac{2 - D}{10} = \frac{2 - 0.75}{10} = .125$$

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

Backoff models: absolute discounting

see the dog	1
see the cat	2
see the banana	4
see the man	1
see the woman	1
see the car	1

p(puppy | see the) = ?

α(see the) = ?

How much probability mass did we reserve/discount for the bigram model?

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

Backoff models: absolute discounting

see the dog	1
see the cat	2
see the banana	4
see the man	1
see the woman	1
see the car	1

p(puppy | see the) = ?

α(see the) = ?

$$\frac{\text{\# of } \textit{types} \text{ starting with “see the” * D}}{\text{count(“see the”)}}$$

For each of the unique trigrams, we subtracted D/count(“see the”) from the probability distribution

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

Backoff models: absolute discounting

see the dog	1
see the cat	2
see the banana	4
see the man	1
see the woman	1
see the car	1

p(puppy | see the) = ?

α(see the) = ?

$$\frac{\text{\# of } \textit{types} \text{ starting with “see the” * D}}{\text{count(“see the”)}}$$

$$reserved\_mass(\text{see the}) = \frac{6 * D}{10} = \frac{6 * 0.75}{10} = 0.45$$

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

distribute this probability mass to all bigrams that we are backing off to

Calculating α

- We have some number of bigrams we're going to backoff to, i.e. those X where C(see the X) = 0, that is unseen trigrams starting with "see the"
- When we backoff, for each of these, we'll be including their probability in the model: P(X | the)
- α is the normalizing constant so that the sum of these probabilities equals the reserved probability mass

$$\alpha(\textit{see the}) \sum_{X:C(\text{see the X}) == 0} p(\text{X} \mid \text{the}) = reserved\_mass(\text{see the})$$

Calculating α

- We can calculate α two ways
  - Based on those we haven't seen:

$$\alpha(\text{see the}) = \frac{reserved\_mass(\text{see the})}{\sum_{X:C(\text{see the X})=0} p(\text{X} \mid \text{the})}$$

  - Or, more often, based on those we do see:

$$\alpha(\text{see the}) = \frac{reserved\_mass(\text{see the})}{1 - \sum_{X:C(\text{see the X})>0} p(\text{X} \mid \text{the})}$$

Calculating α in general: trigrams

- Calculate the reserved mass

$$\text{reserved\_mass(bigram)} = \frac{\text{\# of } types \text{ starting with bigram * D}}{\text{count(bigram)}}$$

- Calculate the sum of the backed off probability. For bigram "A B":

$$1 - \sum_{X:C(\text{A B X})>0} p(\text{X} \mid \text{B}) \qquad \sum_{X:C(\text{A B X})=0} p(\text{X} \mid \text{B})$$

either is fine in practice, the left is easier

- Calculate α

$$\alpha(\text{A B}) = \frac{reserved\_mass(\text{A B})}{1 - \sum_{X:C(\text{A B X})>0} p(\text{X} \mid \text{B})}$$

1 – the sum of the bigram probabilities of those trigrams that we saw starting with bigram A B

Calculating α in general: bigrams

- Calculate the reserved mass

$$\text{reserved\_mass(unigram)} = \frac{\text{\# of } types \text{ starting with unigram * D}}{\text{count(unigram)}}$$

- Calculate the sum of the backed off probability. For bigram "A B":

$$1 - \sum_{X:C(\text{A X})>0} p(\text{X}) \qquad \sum_{X:C(\text{A X})=0} p(\text{X})$$

either is fine in practice, the left is easier

- Calculate α

$$\alpha(\text{A}) = \frac{reserved\_mass(\text{A})}{1 - \sum_{X:C(\text{A X})>0} p(\text{X})}$$

1 – the sum of the unigram probabilities of those bigrams that we saw starting with word A

Calculating backoff models in practice

- Store the αs in another table
  - If it's a trigram backed off to a bigram, it's a table keyed by the bigrams
  - If it's a bigram backed off to a unigram, it's a table keyed by the unigrams
- Compute the αs during training
  - After calculating all of the probabilities of seen unigrams/bigrams/trigrams
  - Go back through and calculate the αs (you should have all of the information you need)
- During testing, it should then be easy to apply the backoff model with the αs pre-calculated

Backoff models: absolute discounting

the Dow Jones	10
the Dow rose	5
the Dow fell	5

p(jumped | the Dow) = ?

What is the reserved mass?

$$\frac{\text{\# of } types \text{ starting with "see the" * D}}{\text{count("see the")}}$$

$$reserved\_mass(\text{the Dow}) = \frac{3*D}{20} = \frac{3*0.75}{20} = 0.115$$

$$\alpha(\text{the Dow}) = \frac{reserved\_mass(\text{see the})}{1 - \sum_{X:C(\text{the Dow X}) > 0} p(\text{X} \mid \text{the})}$$

Backoff models: absolute discounting

$$\text{reserved\_mass} = \frac{\text{\# of } types \text{ starting with bigram * D}}{\text{count(bigram)}}$$

- Two nice attributes:
  - decreases if we've seen more bigrams
    - should be more confident that the unseen trigram is no good
  - increases if the bigram tends to be followed by lots of other words
    - will be more likely to see an unseen trigram

Kneser-Ney

- Idea: not all counts should be discounted with the same value

common

P(Francisco | eggplant) vs
P(stew | eggplant)

rarer

If we've never seen either bigram before, which should be more likely? why?

What would an normal discounted backoff model say?

What is the problem?

Kneser-Ney

- Idea: not all counts should be discounted with the same value

P(Francisco | eggplant) vs
P(stew | eggplant)

Problem:
- Both of these would have the same backoff parameter since they're both conditioning on eggplant
- We then would end up picking based on which was most frequent
- However, even though Francisco tends to only be preceded by a small number of words

Kneser-Ney

- Idea: not all counts should be discounted with the same value
- "Francisco" is common, so backoff/interpolated methods say it is likely
  - But it only occurs in context of "San"
- "stew" is common in many contexts

- Weight backoff by number of contexts word occurs in

P(Francisco | eggplant) low
P(stew | eggplant) higher

Kneser-Ney

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{absolute}(z \mid y) & \text{otherwise} \end{cases}$$

instead of the probability of the word/bigram occurring, use the probability of the word to follow other words

$$P_{absolute}(z \mid xy) = \begin{cases} \frac{C(xyz) - D}{C(xy)} & \text{if } C(xyz) > 0 \\ \alpha(xy) P_{CONTINUATION}(z \mid y) & \text{otherwise} \end{cases}$$

PCONTINUATION

- Relative to other words, how likely is this word to continue (i.e. follow) many other words

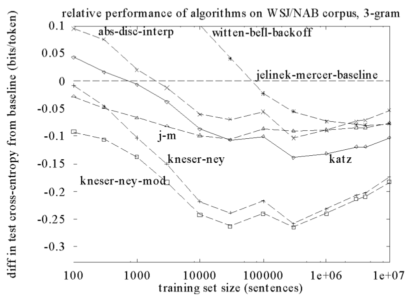
$$P_{CONTINUATION}(z \mid y) = \frac{\#\text{ types ending with } yz}{\sum_{bc \in bigrams} \#\text{types ending with bigram } bc}$$

$$= \frac{|\{xyz : C(xyz) > 0\}|}{\sum_{bc \in bigrams} |\{abc : C(abc) > 0\}|}$$

Other language model ideas?

- Skipping models: rather than just the previous 2 words, condition on the previous word and the 3rd word back, etc.
- Caching models: phrases seen are more likely to be seen again (helps deal with new domains)
- Clustering:
  - some words fall into categories (e.g. Monday, Tuesday, Wednesday…)
  - smooth probabilities with category probabilities
- Domain adaptation:
  - interpolate between a general model and a domain specific model

Smoothing results



Take home ideas

- Key idea of smoothing is to redistribute the probability to handle less seen (or never seen) events
  - Must always maintain a true probability distribution
- Lots of ways of smoothing data
- Should take into account features in your data!
- For n-grams, backoff models and, in particular, Kneser-Ney smoothing work well

Language Modeling Toolkits

- SRI
  - http://www-speech.sri.com/projects/srilm/
- CMU
  - http://www.speech.cs.cmu.edu/SLM_info.html